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Hopf Bifurcation in the Study of Synchronous Motor Stability

Bifurcación de Hopf en el estudio de la estabilidad del motor síncrono

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Abstract

In this work, the dynamic model of the synchronous motor was analyzed, which has a typical structure of Lienard-type systems. For this, the theory of dynamic systems was used, especially the Hopf bifurcation. The objective is to apply this type of bifurcation to the model described in order to show the variations in the equilibrium points of the system by taking as a variable parameter the voltage of the bus to which it is connected. The conditions that the voltage of the infinite bus to which the network is connected must meet in order for it to have asymptotic or spiral stability. It can then be shown that when the bus voltage presents variations, the equilibrium points change their dynamics from asymptotic stability to spiral stability.

Keywords**:** Dynamic systems, Equilibrium points, Periodic orbits, Stable system, Unstable system.

Resumen

En este trabajo se analizó el modelo dinámico del motor síncrono, el cual tiene una estructura típica de los sistemas tipo Lienard. Para ello se utilizó la teoría de los sistemas dinámicos, en especial la bifurcación de Hopf. El objetivo es aplicar este tipo de bifurcación al modelo descrito para mostrar las variaciones en los puntos de equilibrio del sistema tomando como parámetro variable la tensión de la barra a la que está conectado. Las condiciones que debe cumplir la tensión de la barra infinita a la que está conectada la red para que tenga estabilidad asintótica o espiral. Entonces se puede demostrar que cuando la tensión de la barra presenta variaciones, los puntos de equilibrio cambian su dinámica de estabilidad asintótica a estabilidad espiral.

Palabras clave**:** Sistemas dinámicos, Puntos de equilibrio, Orbitas periódicas, Sistema estable.

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1 Introduction

Synchronous motors are highly used in electrical power systems because they are an economical means to improve the power factor of the network, generating a reduction in the cost of electrical energy. This characteristic, together with the ability to operate at constant speeds determined by some types of loads, make synchronous motors indispensable for the industry, with applications in different areas such as: Quarries and cement factories to move ball mills or mills. rollers, compressors, and blowing machines such as extractors, fans, turbofan, centrifugal compressors; in sawmills, in paper mills to move refiners, in rubber and plastic industries to power mixers, steel industries, among many more applications [1-3].

The stability of a dynamic system is the ability of the system to maintain its equilibrium point against certain external disturbances, in electrical power systems these disturbances are due to abnormal situations in operation such as variations in loads, failures caused by natural factors, among many other factors. The treatment of the stability problem consists of establishing those conditions for which the operation of the system (generators, motors, or capacitors) turns out to be critical, that is, limit conditions; in such a way that stability is defined for any other condition [4-6].

The purpose of this document is to analyze in its different aspects, the specific problem of stability in synchronous motor operates, as a device for converting electrical power into mechanical power, since hyperbolicity and stable structure are strongly related, since when there is presence of an eigenvalue with zero real part, the possibility that the system is structurally stable is broken. Thus, through the implementation of the bifurcation theory, it will be in charge of establishing conditions in the parameters of the dynamic system in question for which the system goes from being stable to being unstable. These conditions occur at a specific value of the parameters called the branch point.

2 Theoretical Formalisms

2.1 Lienard Type dynamics model

The dynamics of the synchronous motor represented schematically in the Figure 1 is governed by the second order differential equation (1) [7]. The synchronous motor is connected to an infinite voltage bus V_∞ through a reactance x_L, and is modeled with an FEM E_q and a transient reactance x'_d.

$$P_e - P_m - P_d = H\frac{d^2\delta}{dt^2} \tag{1}$$

Where δ is the rotor angle of the synchronous motor, H is the constant inertia, P_e, P_m, and P_d are the electrical, mechanical and damping powers respectively, associated with the motor dynamics and defined as:

$$\begin{aligned} P_e &= c_1\sin(\delta) - c_2\sin(2\delta) \\ P_m &= c_1\sin(\delta_0) - c_2\sin(2\delta_0) \\ P_d &= c_3\sin^2(\delta) + c_4\cos^2(2\delta) \end{aligned} \tag{2}$$

The load P_m in the synchronism is assumed constant,

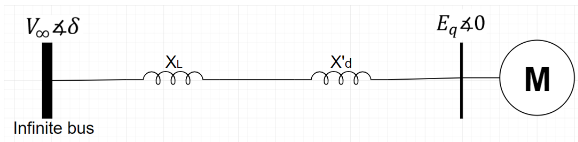

Figure 1. Synchronous motor diagram

independent of the small variations in speed during external disturbances. Furthermore, the model parameters c_1, c_2, c_3, and c_4 are defined by Equation (3) [7] as:

$$\begin{aligned} c_1 &= \frac{V_\infty E_q}{x_L + x'_d} \\ c_2 &= \frac{V_\infty^2(x_q - x'_d)}{2(x_L + x_q)(x_L + x'_d)} \\ c_3 &= \frac{V_\infty^2(x'_d - x''_d)T''_{d0}}{(x_L + x'_d)^2} \\ c_4 &= \frac{V_\infty^2(x'_q - x''_q)T''_{q0}}{(x_L + x'_q)^2} \end{aligned} \tag{3}$$

The parameters c_1, c_2 are the amplitude of the fundamental component and the second harmonic of the electrical power and the parameters c_3, c_4 decide on the amplitude of the damping power P_d imposed on the rotor of the synchronous motor. The constant damping case can be analyzed when $c_3 = c_4$. The steady state equilibrium point of the synchronous motor is given by $\delta = \delta_0$ and $\dot{\delta} = 0$ [7-10]. If we define the state variables $x_1 = \delta(t)$ and $x_2 = \frac{d\delta(t)}{dt}$ for model gyven by the Equation (1), the following system of differential equations is obtained in the state space (Equation 4) [9]:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= K_2(\sin 2\delta_0 - \sin 2x_1) - K_1(\sin\delta_0 - \sin x_1) \\ &\quad - (K_3 \sin^2 x_1 + K_4 \cos^2 x_1)\cdot x_2 \end{aligned} \quad (4)$$

Being $K_i = \frac{c_i}{H}$ for $i = 1,2,3,4$. The model in state space represented in Equation (4), has the form of a Lienard-type system which is defined by Equation (5) [9]:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -g(x_1) - f(x_1)\cdot x_2 \end{aligned} \quad (5)$$

resulting from the second-order differential equation $x'' + f(x)x' + g(x) = 0$. This type of system has the characteristic of presenting limit cycles if the functions $f(x)$ and $g(x)$ meet certain conditions, which are stated in the following Lienard theorem.

2.2 Lienard's theorem

The system $f(\mathbf{x}, \mu) = 0$, has a single stable limit cycle around the origin if the functions $f(x)$ and $g(x)$ satisfy the following conditions:

- $f(x)$ and $g(x)$ must be continuously differentiable for every value of x.
- $g(-x) = -g(x)$ for all x, that is, it must be an odd function.
- $g(x) > 0$ for all $x > 0$.
- $f(-x) = f(x)$ for all x.
- The odd function $F(x) = \int_0^x [f(u)du]$ has exactly one zero at $x = a$, being negative for $0 < x < a$, positive and non-decreasing for $x > a$ and $F(x) \longrightarrow \infty$ when $x \longrightarrow \infty$.

2.3 Foundation of nonlinerar dynamics systems

The mathematical model of a dynamic system can be represented in state space by a set of first-order differential equations as observed in Equation (6) [11]:

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mu) \quad (6)$$

where $\mathbf{x} \in \mathbb{R}^n$, being $\mathbf{x}$ is the vector of states of the system and $\mu \in \mathbb{R}^k$ with μ as the state parameter. The solution $x(t)$ of the system gyven by the Equation (6) will depend on the initial conditions defined. For nonlinear dynamic systems, these equations are calculated mostly by computational integration techniques [11]. The qualitative analysis is strictly related to the projection of the trajectories $\mathbf{x}(t)$ in the phase space, called phase portrait. In this diagram all the qualitative characteristics of the behavior of the system are observed, one of the most relevant is the analysis of the isolated equilibrium points since there is sufficient mathematical background that allows a clear classification of these points, on the other hand, It should be noted that for non-linear dynamic systems the equilibrium points are not unique and depending on the initial conditions the system reaches different equilibria.

2.4 Equilibrium points

The equilibrium points for the system represented by Equation (6) are given by $\dot{\mathbf{x}} = 0$, that is; by Equation (7) [11]:

$$f(\mathbf{x}, \mu) = 0 \quad (7)$$

For a given value of μ, the solution of Equation (7) will be an equilibrium point for the system given by Equation (6). Once the equilibrium points have been defined, it is necessary to study the classification of their stability through linearization around them, as shown.

Taking $\mathbf{x}_o$ y μ_0 as equilibrium points, the system gyven by Equation (6) can be rewritten as Equation (8) [11]:

$$\dot{\mathbf{x}} = A\mathbf{x} \quad (8)$$

where A is the Jacobian matrix of the system represented by Equation (1), defined according to Equation (9) [11]:

$$A = [a_{ij}] = \left[\frac{\partial f_i}{\partial x_j}\right] with \ \ \mathbf{x} = \mathbf{x}_o$$

The stability of each of the equilibrium points of the system represented by Equation (6) will depend on the eigenvalues of matrix A; They can also be classified depending on the values taken by the trace of matrix $A(Tr(A))$ and its determinant $(Det(A))$. A summary of this is graphically shown in Figure 2 [12, 13].

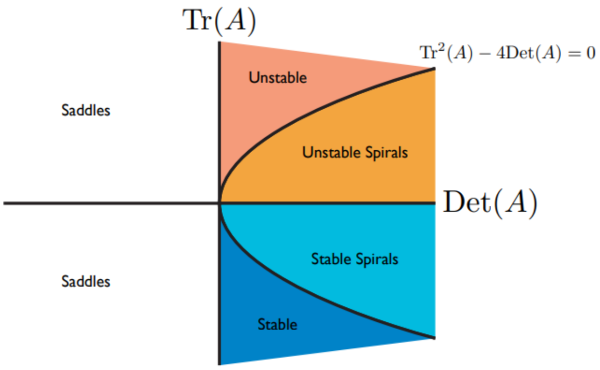

Figure 2. Classification of the equilibrium points according to $Tr(A)$ and $Det(A)$.

2.5 Hyperbolicity and structural stability

One of the most relevant properties that can be obtained from the eigenvalues of matrix A to characterize the equilibrium point analyzed is the concept of hyperbolicity. If none of the eigenvalues of matrix A has a real part zero, the point x_0 is said to be a hyperbolic point. Two consequences of a hyperbolic equilibrium point are the following:

- If matrix A has no zero eigenvalues, then x_0 is a simple transverse zero of (7). Therefore, by the implicit function theorem, the existence of a smooth function $x(\mu)$ with $x(\mu_0) = x_0$ that shows the variation of x_0 when the parameter μ is varied is guaranteed. Furthermore, there is no variation in the number of equilibrium points when $\mu = \mu_0$.

- The qualitative structure of the phase diagram for the non-linear system is the same as that of the linearized system, this as a consequence of the Hartman-Grodman theorem [14, 15]. This fact is very important from the qualitative characterization of a non-linear system through its linearization. It should be noted that, for non-hyperbolic equilibrium points, the analysis of their stability will not be the same for both systems, that is, it is not possible to conclude about their stability through the criterion of eigenvalues and it is necessary to study by other techniques such as the construction of energy functions [16, 17].

A system can be robust if when making small modifications of a parameter, the trajectories of the phase space are only slightly disturbed, if this is the case, the system is said to be structurally stable. Both concepts of hyperbolicity and stable structure are strongly related since when there is the presence of an eigenvalue with zero real part, the possibility that the system is structurally stable is broken.

2.6 Bifurcation Theory

Bifurcation theory is a branch of applied mathematics where its main interest is the analysis of the Equation (7), where x is an equilibrium solution and μ is a scalar parameter, that is, determining how is the variation of $x(\mu)$ for when μ varies. The parameter μ is called the branch parameter. A bifurcation point is one where there is a branching or change of the solution. System stability is closely related to this bifurcation phenomenon [18].

2.7 Hopf bifurcation

The Hopf bifurcation is also known as the Poincaré-Hopf-Andronov bifurcation, it is characterized mainly by the appearance or disappearance of a periodic solution (limit cycle) of an equilibrium when the μ parameter of the system represented by Equation (6) causes the eigenvalues to cross the axis imaginary from left to right. There are two types of Hopf bifurcation, supercritical or subcritical, stable, or unstable within a manifold, respectively.

2.8 Hopf theorem

Taking the following considerations for the system described in Equation (6):

1. The system has an equilibrium point at $P_0 = (\chi_0, \mu_0)$

2. The Jacobian matrix of (6) has a conjugate pair of eigenvalues $\lambda = \alpha \pm j\omega$ such that for

a critical value of the bifurcation parameter $\mu = \mu_c$ it holds that $\alpha(\mu_c) = 0$, $\alpha'(\mu_c) \neq 0$ and $\omega(\mu_c) > 0$ where $\alpha' = \frac{d\alpha}{d\mu}$

3. Except for $\pm j\omega(\mu_c)$ the eigenvalues of the system have a negative real part.

If these conditions are fulfilled, then for a disturbance in the state x_i, the dynamics of the system represented by Equation (6) has a stationary solution. The dynamics of the system corresponding to the stationary solution will be stable for $\mu_c - \mu > 0$ and unstable for $\mu_c - \mu < 0$ if $\alpha'(\mu_c) > 0$. Stable for $\mu_c - \mu < 0$ and unstable for $\mu_c - \mu > 0$ if $\alpha'(\mu_c) < 0$. For $\mu = \mu_c$ the system will have oscillations with period $2\pi/\omega(\mu_c)$ [19].

3 Simulation and results

For the results obtained in this section, the following values of the parameters were used:

$$X_d = 1.7\ ;\ X_d' = 0.245\ ;\ X_d'' = 0.185$$
$$X_q = 1.64\ ;\ X_q' = 0.38\ ;\ X_q'' = 0.18\ ;\ X_L = 0.3$$
$$T_{d0}'' = 5.9\ ;\ T_{q0}'' = 0.075\ ;\ H = 0.3108$$
$$E_q = 0.887\ ;\ V_\infty = 1$$

The equilibrium points of the system represented by Equation (4) are given by the Equation (10):

$$\begin{aligned} & x_2 = 0 \\ & K_2(sin2\delta_0 - sin2x_1) - K_1(sin\delta_0 - sinx_1) \\ & -(K_3 sin^2 x_1 + K_4 cos^2 x_1)\cdot x_2 = 0 \end{aligned} \tag{9}$$

With which the point $P_e(\delta_0, 0)$ is obtained and its associated Jacobian matrix will be represented by Equation (11):

$$\begin{bmatrix} 0 & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{10}$$

where $a_{12} = 1$, $a_{21} = K_1\cos(\delta_0) - 2K_2\cos(2\delta_0)$ and $a_{22} = -(K_3\sin^2(\delta_0) + K_4\cos^2(\delta_0))$. Therefore, the trace and determinant will be defined as:

$$\begin{aligned} Tr(J) &= -(K_3 sin^2(\delta_0) + K_4 cos^2(\delta_0)) \\ Det(J) &= K_1 cos(\delta_0) - 2K_2 cos(2\delta_0) \end{aligned} \tag{11}$$

For the parameters of the previous table, we have that $Tr(J) < 0$ and that $Det(J) < 0$, with which it is obtained that the equilibrium point of interest is a saddle point as classified in the Figure 3. In addition, two more points are obtained which correspond to sinks, with which the stability of the system will be determined not only by the value of its parameters, but also by the initial conditions taken. By varying the magnitude of the bus voltage to

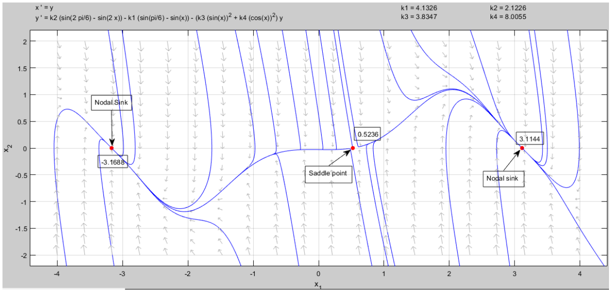

Figure 3. Phase space x_1 vs x_2 asymptotic stability.

which the synchronous motor is connected, there is a variation in the stability of the equilibrium points, which go from having asymptotic stability to having spiral stability as shown in the Figure 4. When a

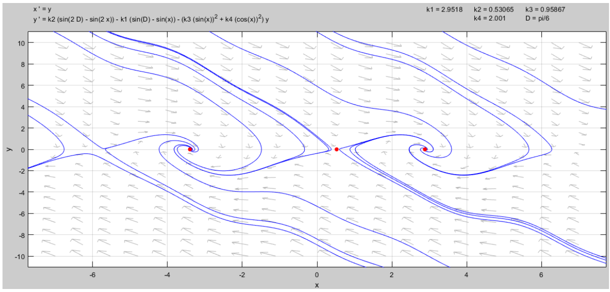

Figure 4. Spiral stability of balance points.

constant damping value is taken equal to zero, that is $K_3 = K_4 = 0$, it is given that the dynamics of the stable points (sinks) become center points, as observed in the Figure 5.

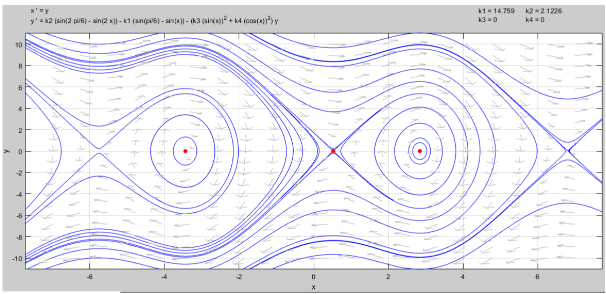

Figure 5. Phase space x_1 vs x_2 with zero damping.

4 Conclusions

Qualitative analysis for nonlinear dynamic systems is of vital importance since in most of these cases the analytical solutions are impossible to determine. Besides, with the bifurcation theory, critical values can be established in the parameters where the stability of the system will present variations. Also, an application of the Hopf bifurcation theorem was presented for the case of an asynchronous motor with variable damping. The conditions that the voltage of the infinite bus to which the network is connected must meet for it to have asymptotic or spiral stability were established. It was determined that when the bus voltage presents variations, the equilibrium points change their dynamics from asymptotic stability to spiral stability. When the synchronous motor enters the instability state, there is only one option for this, and it is the asymptotic type of instability since spiral type instability cannot occur. For the parameters defined in this application, it was observed that $Tr(J) < 0$ and that $Det(J) < 0$, that is, the equilibrium point of interest is a saddle point. When a constant damping value is taken equal to zero, this is $K_3 = K_4 = 0$, it is given that the dynamics of the stable points (sinks) become center points

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