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Circuit model characterization of shunt reactors using a nonlinear programming model

Caracterización del modelo de circuito de reactores de derivación utilizando un modelo de programación no lineal

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Abstract

This research presents the development of an optimization model to estimate parameters for reactive power compensators in medium voltage networks using reactors. The proposed mathematical modeling belongs to the family of nonlinear programming models. The proposed mathematical model considers multiple measures regarding applied voltage in terminals of the reactor as well as data regarding active and reactive power behavior and input current. The objective function considered corresponded to the minimization of the mean square error between the measured and calculated variables. To solve the proposed optimization model is employed the General Algebraic Modeling System (GAMS) software. Numerical results in two reactors with nominal compensation capabilities of about 2 Mvar and 6.75 Mvar, operated with 13, and 25 kV, demonstrate the effectiveness of the proposed optimization model to characterize the electrical circuit of these compensation devices. Different nonlinear programming algorithms available in GAMS were employed in the solution of the proposed optimization model with objective functions lower than 1×10^{-10}, which confirms that the measured and calculated variables have the same numerical behavior, which allows concluding that the characterized circuit reflects the expected electrical behavior of the reactors under different voltage input.

Keywords: Medium-voltage distribution networks; reactors; circuit model characterization; parameter estimation; nonlinear programming model; GAMS software

Resumen

Esta investigación presenta el desarrollo de un modelo de optimización para estimar parámetros para compensadores de potencia reactiva en redes de media tensión utilizando reactores. El modelado matemático propuesto pertenece a la familia de modelos de programación no lineal. El modelo matemático propuesto considera múltiples medidas con respecto al voltaje aplicado en las terminales del reactor, así como datos con respecto al comportamiento de la potencia activa, reactiva y la corriente de entrada. La función objetivo considerada correspondió a la minimización del error cuadrático medio entre las variables medidas y calculadas. Para resolver el modelo de optimización propuesto se emplea el software General Algebraic Modeling System (GAMS). Los resultados numéricos en dos reactores con capacidades nominales de compensación de alrededor de 2 y 6.75 Mvar, operados con 14 y 25 kV, demuestran la efectividad del modelo de optimización propuesto para caracterizar el circuito eléctrico de estos dispositivos de compensación. En la solución del modelo de optimización propuesto con funciones objetivo menores a 1×10^{-10} se emplearon diferentes algoritmos de programación no lineal disponibles en GAMS, lo que confirma que las variables medidas y calculadas tienen el mismo comportamiento numérico, lo que permite concluir que el circuito caracterizado refleja el comportamiento eléctrico esperado de los reactores bajo diferentes voltajes de entrada.

Palabras Clave: Redes de distribución de media tensión; reactores; caracterización del modelo de circuito; estimación de parámetros; modelo de programación no lineal; software GAMS

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Nomenclature

Parameters

μ Permeability of the ferromagnetic material (Hm).

K Number of measurement.

L_s Inductance of the the winding (H).

N Number of winding turns.

R_M Resistive effect of the ferromagnetic core (Ω).

$R_M^{\min}$, $R_M^{\max}$ Minimum and maximum resistance of the shunt winding (Ω).

R_s Resistive of the the winding (Ω).

$R_s^{\min}$, $R_s^{\max}$ Minimum and maximum resistance of the series winding (Ω).

X_M Reactance of ferromagnetic core (Ω).

$X_M^{\min}$, $X_M^{\max}$ Minimum and maximum reactance of the shunt winding (Ω).

X_s Reactance of the winding (Ω).

$X_s^{\min}$, $X_s^{\max}$ Minimum and maximum reactance of the series winding (Ω).

Variables

$\bar{I}_{r,k}$ Measured current input in shunt reactor (A).

$\bar{P}_{r,k}$ Measured active power input in shunt reactor (W).

$\bar{Q}_{r,k}$ Measured reactive power input in shunt reactor (Var).

$\mathbf{I}_r$ Current flowing through the winding in phasor form (A).

$\mathbf{V}_M$ Voltage drop on magnetization branch in phasor form (V).

$\mathbf{V}_r$ Voltage input in phasor form (V).

$\Phi_B(t)$ Magnetic flux on the ferromagnetic core (Wb).

$\mathbf{Z}_r$ Impedance of the reactor (Ω).

$B(t)$ Magnetic field density (T).

$H(t)$ Magnetic field intensity (Am).

i_r Current flowing through the winding (A).

k kth measure.

$v(t)$ External voltage input (V).

v_r Sinusoidal voltage input (V).

Z_r magnitude of the impedance $\mathbf{Z}_r$ (Ω).

1 Introduction

The shunt reactor devices have received great reverence in the operation of the electrical system since they can compensate for the capacitive reactive power that provides essential facts, such as voltage control, power system stability, improved power quality, cost savings, and power loss reduction [1]–[3]. The voltage control is performed when the electrical system has a high capacitance reactive power, which can cause overvoltage and instability problems [4]. Thus, the shunt reactor device alleviates these problems, absorbing excess reactive power. In the case of power system stability problems, the shunt reactor device has an essential role in compensating for sudden changes in generation/load, leading to an unbalance of reactive power [5]. This unbalance can generate voltage stability problems. Hence, the shunt reactor devices quickly absorb or inject large amounts of reactive power. For the improvement of the power quality, the shunt reactor devices can reduce harmonics, voltage fluctuations, and other events that can cause some critical equipment in the electrical system to malfunction [6]. At the distribution level, the shunt reactor devices are usually installed at electrical distribution substations or the end of an extended electrical distribution branch. They help to regulate the voltage profiles, mitigating some voltage fluctuations that distributed generators can generate [3], [7]. Furthermore, the shunt reactor device decreases power losses in the distribution network and thus enhances the power factor by reducing the currents required. Resulting of the above, the energy operating costs in the distribution network are reduced [7].

On the other hand, having a mathematical model of a shunt reactor device and its parameters is crucial to accurately analyze and simulate its behavior and impact on an electrical system. The mathematical model that represents the shunt reactor device's behavior and parameters comprises its inductive, resistant, and capacitive effects, as well as its nominal voltage and nominal current [8]. With this model, it is possible to compute the reactive power compensation by the shunt reactor devices and their impact on the voltage and current in the distribution network. Therefore, knowing the parameters of the shunt reactor model is crucial to guarantee the satisfactory operation of the electrical distribution system [7].

In the current literature, the problem of estimating parameters in electrical devices has been widely explored. Multiple research has presented mathematical models, and solution methodologies for electrical machines [9], and photovoltaic sources [10]. In the case of the electrical transformers, authors of [11] presented the nonlinear programming model (NLP) for the parameter estimation in single-phase transformers using voltage and current measures. The General Algebraic Modeling System (GAMS) software solved the proposed nonlinear programming model. In Ref. [12], the chaotic search algorithm has been proposed to solve the NLP model for parametric estimation in single-phase transformers. The authors' main contribution is developing an experimental validation considering online measures to verify the efficiency of the proposed estimation approach. In the case of induction machines, the parametric estimation problem has been addressed with combinatorial optimization methods by considering multiple measures of the torques and power factor in different operative conditions. Some of the optimization algorithms applied are particle swarm optimization [13], genetic algorithms [14], polynomial regression [15], and differential evolution algorithm [9]. In the case of photovoltaic modules, as happened with the transformers and induction motors, the estimation problem is formulated using an NLP model. Different metaheuristic algorithms have been applied to identify the electrical parameters of these photovoltaic sources. These algorithms include the grasshopper optimization algorithm [10], the sine cosine algorithm [16], and the continuous genetic algorithm [17], [18], among others.

Table 1 summarizes the main literature approaches for estimating parameters in transformers and induction motor.

Regarding reactors, in the scientific literature, the analysis of reactors is mainly focused on their electromagnetic design by selecting the best ferromagnetic material and the geometric characteristics of the core by using finite element analysis and design [30], [31]. In Ref. [8] proposed a harmonic-based approach for analyzing and characterizing a reactor for extra-high voltage application considering its nonlinear core behavior. Numerical results confirm that the electrical behavior of the reactor is linear in the region of operation near the design voltage, which was confirmed with advanced simulations using specialized electromagnetic analysis software. Owing to the limitations in analyzing and characterizing the electrical circuit that represents the electrical behavior of shunt reactors for distribution network applications. This research presents the following contributions:

i. The formulation of an NLP model to represent the circuit model characterization of shunt reactors for medium voltage (MV) applications using an equivalent circuit with four main parameters, i.e., series and parallel resistances and reactances, where the series elements model the winding wire effects, and parallel branch models the ferromagnetic core.

ii. The solution of the proposed NLP model minimizes the mean square error between the calculated and measured electrical variables considering the constraints associated with the equivalent impedance and the nonlinear relations between the input current and the active and reactive power variables using different GAMS solvers.

Remark 1 *In Table 1, the main characteristic of all the optimization methods applied to the problem of parametric estimation in transformers and induction machines is that all of them deal with solving NLP formulations, where the primary objective is minimizing the mean square error between the measured and calculated variables subject to the electrical model of the device under analysis.*

It is worth mentioning that this research is considered MV applications; the shunt reactor operates in their linear region of the magnetization curve, i.e., the parallel branch is modeled with constant resistive and reactance parameters without including the saturation effects. In addition, the main result of this research will be the confirmation that in the case of the parametric estimation for shunt reactors, it is presented a multi-modal behavior, i.e., the existence of multiple solutions with the same objective function value. Two MV reactors demonstrate this multi-modal behavior by solving the NLP model with different GAMS solvers.

The remainder of this document is structured as follows: Section 2 presents the general model of a reactor for steady-state analysis through an equivalent electrical circuit that assumes that under nominal operating conditions, the reactor is operating in the linear zone of the magnetization curve, i.e., no saturation effects on the ferromagnetic core are considered. Section 3 describes the general nonlinear programming model that allows characterizing the electrical behavior of a reactor in MV applications considering multiple voltages, current and power measures. The presented optimization model minimizes the mean square error between the measured and calculated variables subject to Kirchhoff's law and Tellegens' theorem. Section 4 reveals the main characteristics of the solution methodology by using the GAMS software. This section is described through a flow diagram the main aspects regarding the solution of a nonlinear programming model in the GAMS software. Section 5 shows the main characteristics of the test reactors, which correspond to two reactors for MV applications, the first reactor is designed for operating with 13 kV and 2 Mvar, and the second reactor operates with 25 kV and absorbs 6.75 Mvar. Section 6 presents the simulation results for both test reactors in the GAMS software with three different NLP solvers available (i.e., CONOPT, COUENNE, and IPOPT). Finally, Section 7 lists the main concluding remarks derived from this work and some possible future research.

2 Electrical model of a shunt reactor

A shunt reactor is an electrical machine composed mainly of a winding of copper wire on a ferromagnetic core, where the inductive effect is the predominant electrical parameter [8]. The schematic representation of a reactor is depicted in Figure 1.

The mathematical model of a shunt reactor can be represented as follows:

$$v_r(t) = R_s i_r(t) + L_s \frac{d}{dt} i_r(t) + N \frac{d}{dt} \Phi_B(t) \qquad (1)$$

where $v_r(t)$ is the sinusoidal voltage input, R_s and L_s represent the resistive and inductive effects on the winding, N is the number of winding turns, and $\Phi_B(t)$ is the magnetic

Tabla 1: Solution methodologies applied to the parametric estimation in electrical machines

Device	Solution methodology	Year	Ref.
Transformers	Bacterial foraging algorithm	2010	[19]
	Chaotic optimization algorithm	2019	[12]
	Exact NLP model solved in GAMS	2020	[11]
	Manta ray foraging optimization and chaotic manta ray foraging optimization methods	2020	[20]
	Jellyfish search optimizer algorithm	2021	[21]
	Black hole optimization	2021	[22]
	Sine cosine algorithm	2021	[23]
	Crow search algorithm	2022	[24]
	Gravitational search algorithm	2023	[25]
Induction motors	Particle swarm optimization	2010	[13]
	Genetic algorithm combined with particle swarm optimizer	2014	[14]
	Charged system search and differential evolution algorithm	2014	[9]
	Two-stage recursive least squares method	2015	[26]
	Mean squares method	2015	[27]
	Polynomial regression	2018	[15]
	Differential evolution algorithm	2018	[28]
	Simplified search algorithm based on the Thevenin equivalent	2020	[29]

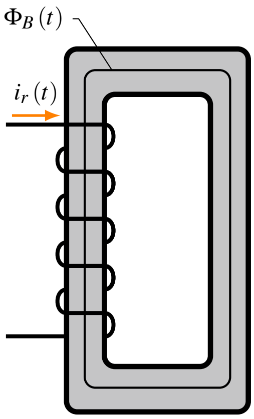

Figure 1: Electrical wire wound on a ferromagnetic core

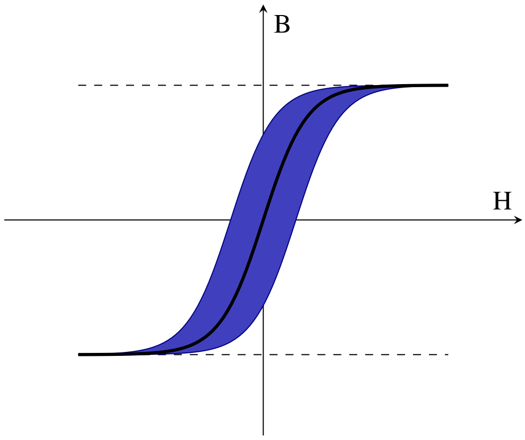

Figure 2: Behavior of the B-H curve for a typical ferromagnetic material.

flux on the ferromagnetic core. Note that $v(t)$ is the external voltage input, and $i_r(t)$ is the total current flowing through the winding. In addition, if we assume that the shunt reactor operates in the linear region (see Figure 2). Note that in the linear zone of operation $B(t) = \mu H(t)$, where $B(t)$ is the magnetic field density, $H(t)$ is the magnetic field intensity, and μ is the permeability of the ferromagnetic material.

Then, the electrical circuit that represents the steady-state operative conditions of a reactor for distribution network applications can be observed in Figure 3. Where R_s is the resistive effect of the wire, $X_s = \omega L_s$ is the reactance value associated with the conductor inductance L_s at the ω frequency; R_M is the cumulative resistive effect of the ferromagnetic core that models the energy losses associated with the hysteresis curve (see Figure 2), and the parasitic currents, among others; and $X_M = \omega L_M$ is the inductive effect caused by the ferromagnetic core with a μ permeability, which is also depending on the number of wound turns and the average length and the transversal area section of the ferromagnetic core. Note that $\mathbf{V}_r$ is the voltage phasor that represents the applied voltage to the reactor terminals; $\mathbf{I}_r$ is the phasor associated with the current flow through the reactor wound, and $\mathbf{V}_M$ is the expected voltage drop in the magnetizing branch of the reactor, respectively.

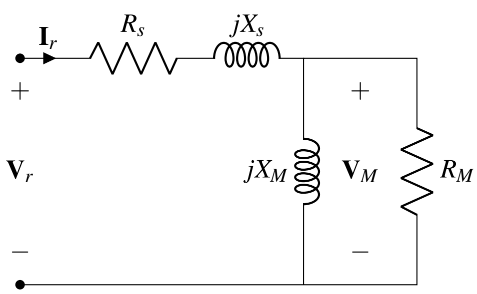

Figure 3: Equivalent circuit of a single-phase reactor.

3 Nonlinear programming model for shunt reactor circuit characterization

In this section, it is presented the general nonlinear programming model that represents the equivalent electrical circuit parametrization of a shunt reactor considering three main measures: (i) the magnitude of the current flow through the shunt reactor wound, (ii) the active power losses in terminals of the reactor, and (iii) the amount of reactive power absorption with the reactor. Note that all of these measures consider different voltage inputs. For the mathematical formulation, let us consider that: $I_{r,k}$ and $V_{r,k}$ are the magnitudes of the electrical current in the wound shunt reactor and the applied voltage during the k^{th} measuring event. In addition, $P_{r,k}$ and $Q_{r,k}$ are the measures of the active and reactive power characteristics of the shunt reactor during the k^{th} measuring event.

3.1 Objective function formulation

The proposed objective function, as typical in parameter estimation models for induction machines [15] or single-phase transformers [12], corresponds to the minimization of the mean square error between the measured and calculated variables [20], [24]. The general structure of the objective function is presented below.

$$\mathrm{MSE} = \frac{1}{2}\sum_{k=1}^{K}\left(\begin{array}{c}\left(\frac{I_{r,k}-\bar{I}_{r,k}}{\bar{I}_{r,k}}\right)^2 + \left(\frac{P_{r,k}-\bar{P}_{r,k}}{\bar{P}_{r,k}}\right)^2 + \\ \left(\frac{Q_{r,k}-\bar{Q}_{r,k}}{\bar{Q}_{r,k}}\right)^2\end{array}\right), \quad (2)$$

where $\bar{I}_{r,k}$, $\bar{P}_{r,k}$ and $\bar{Q}_{r,k}$ correspond to the measured current, active and reactive powers in terminals of the shunt reactor. Note that the main characteristic of the objective function in (2) is that the global optimum is zero since it is a quadratic convex function.

3.2 Set of constraints

The objective function defined in (2) is constrained by the expected electrical behavior of the equivalent electric circuit of the reactor depicted in Figure 3. In this sense, to obtain all the constraints of the model circuit characterization of reactors considering electric measures in their terminals, Ohm's and Kirchhoff's' laws are applied to this circuit.

The electrical impedance of the reactor in Figure 3, i.e., $\mathbf{Z}_r$, is defined y the sum of the series resistive and inductive effect in series with the parallel effect of the magnetization branch. This impedance is defined in Equation (3).

$$\mathbf{Z}_r = R_s + jX_s + j\frac{R_M X_M}{R_M + jX_M} \quad (3)$$

The magnitude of the impedance $\mathbf{Z}_r$ is defined as Z_r and can be obtained by separating Equation (3) in its real and imaginary parts. This magnitude is defined below.

$$Z_r = \sqrt{\left(R_s + \frac{R_M X_M^2}{R_M^2 + X_M^2}\right)^2 + \left(X_s + \frac{R_M^2 X_M}{R_M^2 + X_M^2}\right)^2}. \quad (4)$$

Once the magnitude of the impedance is defined through (4), then the calculated current can be obtained from (5).

$$I_{r,k} = Z_r^{-1} V_{r,k}. \quad (5)$$

Now, to determine the reactor's active and reactive power behavior, it is required to calculate the voltage drop in the parallel branch, i.e., $\mathbf{V}_{M,k}$. This variable is obtained from the electrical circuit in Figure 3 by applying a voltage divisor, as presented in (6).

$$\mathbf{V}_{M,k} = \frac{j\frac{R_M X_M}{R_M + jX_M}}{\mathbf{Z}_r}\mathbf{V}_{r,k}, \quad (6)$$

where its magnitude is defined below.

$$V_{M,k} = \frac{R_M X_M}{Z_r\sqrt{R_M^2 + X_M^2}} V_{r,k}. \quad (7)$$

Note that with the voltage across the parallel impedance, and the current input to the shunt reactor, it is possible to define the active and reactive power calculated as described in (8) and (9).

$$P_{r,k} = R_s I_{r,k}^2 + R_M^{-1} V_{M,k}^2, \quad (8)$$

$$Q_{r,k} = X_s I_{r,k}^2 + X_M^{-1} V_{M,k}^2. \quad (9)$$

Finally, to limit the values of the decision variables to typical values found in MV reactors, the following box-type constraints are added to the optimization model.

$$R_s^{\min} \le R_s \le R_s^{\max}, \quad (10)$$

$$R_M^{\min} \le R_M \le R_M^{\max}, \quad (11)$$

$$X_s^{\min} \le X_s \le X_s^{\max}, \quad (12)$$

$$X_M^{\min} \le X_M \le X_M^{\max}. \quad (13)$$

Observe that using these lower and upper limitations to the decision variables ensures an excellent parametric estimation by reaching typical values in reactors, i.e., these bounds are defined by considering typical data provided by reactors' manufacturers [4].

Remark 2 *The complete nonlinear programming model that represents the circuit model characterization of the shunt reactor is the objective function in (2), which must be minimized, subject to the set of constraints (4), 5, and (7)–(11). Note that the solution space in this optimization model defines a non-convex set, which requires efficient numerical methods to deal with a high-quality solution.*

4 Solution methodology

Owing to the complexities of the optimization model (non-linearities and non-convexities), this research selects the GAMS software to solve this optimization problem [32]. This software is selected since it has been widely used in literature to test new mathematical optimization models efficiently, mainly when they correspond to nonlinear programming problems [11]. Some applications where the GAMS software has been used with excellent numerical results include: the assignment problem [33], the problem of the optimal placement and sizing of dispersed generation [34], [35], the optimal design of the water recycling process and reusability of multiproduct batch plant [34], and the optimal operation of hybrid energy networks with multiple distributed energy resources [36], among others. The main advantages of using the GAMS software to solve complex optimization problems include:

i. It is an interpreted mathematical programming language that permits the researcher to concentrate their attention on the mathematical modeling itself and not on the solution technique [32].

ii. The same mathematical structure used to present the optimization problem is used in the GAMS programming environment, which makes an easy transition between the mathematical formulas and the software solution environment [11].

iii. The GAMS software allows the scalability of the optimization problem by using sets to loop through all data without modifying the mathematical structure of the optimization model [35].

The general implementation of an optimization model in the GAMS software is depicted in the flow diagram in Figure 4 [37].

Remark 3 *For more details regarding the implementation of different optimization models in the GAMS programming environment, the reference [32] can be consulted.*

5 Test systems

In this section, the information of two shunt reactors operated with 14 , 25 kV and 2 , and 6.75 MVA are presented.

5.1 First reactor

This is a shunt reactor connected to a distribution substation with the possibility of compensating 2 Mvar when operated at its nominal voltage, i.e., 13 kV. The list of measures for this shunt reactor, i.e., applied voltage, input current, and active and reactive power behaviors, are listed in Table 2.

The expected design parameters for this shunt reactor, i.e., the upper and lower bounds for this test reactor, are listed in Table 3.

5.2 Second reactor

This shunt reactor is connected to a distribution substation with the possibility of compensating 6.75 Mvar when operated at its nominal voltage, i.e., 25 kV. The list of measures for this shunt reactor, i.e., applied voltage, input current, and active and reactive power behaviors, are listed in Table 4.

The expected design parameters for this shunt reactor, i.e., the upper and lower bounds for this test rector, are listed in Table 5.

Tabla 2: Measured that for the 13 kV, 2 Mvar reactor

$V_{r,k}$ (V)	$I_{r,k}$ (A)	$P_{r,k)}$ (W)	$Q_{r,k}$ (var)
11700	138.503	39725.592	1620000.914
11830	140.042	40613.288	1656200.935
11960	141.581	41510.792	1692800.956
12090	143.120	42418.105	1729800.976
12220	144.659	43335.226	1767200.998
12350	146.198	44262.157	1805001.019
12480	147.737	45198.896	1843201.040
12610	149.276	46145.444	1881801.062
12740	150.815	47101.801	1920801.084
12870	152.354	48067.967	1960201.106
13000	153.892	49043.941	2000001.129
13130	155.431	50029.724	2040201.152
13260	156.970	51025.316	2080801.175
13390	158.509	52030.717	2121801.198
13520	160.048	53045.927	2163201.221
13650	161.587	54070.945	2205001.245

Tabla 3: Lower and upper bounds for the decision variables in the 13 kV, 2 Mvar reactor

Parameter	**Min. Value** (Ω)	**Max. Value** (Ω)
R_s	0.400	0.900
X_s	2.500	4.500
R_M	4000	7000
R_s	65	90

Tabla 4: Measured that for the 25 kV, 6.75 Mvar reactor

$V_{r,k}$ (V)	$I_{r,k}$ (A)	$P_{r,k)}$ (W)	$Q_{r,k}$ (var)
18750	202.538	73541.785	3796875.312
20000	216.041	83674.209	4320000.355
21250	229.543	94460.338	4876875.401
22500	243.046	105900.171	5467500.449
23750	256.548	117993.709	6091875.501
25000	270.051	130740.952	6750000.555
26250	283.553	144141.899	7441875.612
27500	297.056	158196.552	8167500.671
28750	310.558	172904.909	8926875.734
30000	324.061	188266.970	9720000.799
31250	337.563	204282.737	10546875.867

Tabla 5: Lower and upper bounds for the decision variables in the 25 kV, 6.75 Mvar reactor

Parameter	**Min. Value** (Ω)	**Max. Value** (Ω)
R_s	0.250	2.000
X_s	2.000	10.000
R_M	5000	15000
R_s	50	125

6 Numerical results

The computational implementation of the proposed NLP model to characterize the electrical circuit of shunt reactors for MV applications is made in the GAMS software with three NLP solvers, i.e., CONOPT, COUNNE, and IPOPT. These implementations were made on a PC (64-bit version of Microsoft Windows 10 Single Language) with an AMD Ryzen 7 3700 with a 2.3 GHz processor and 16.0 GB RAM.

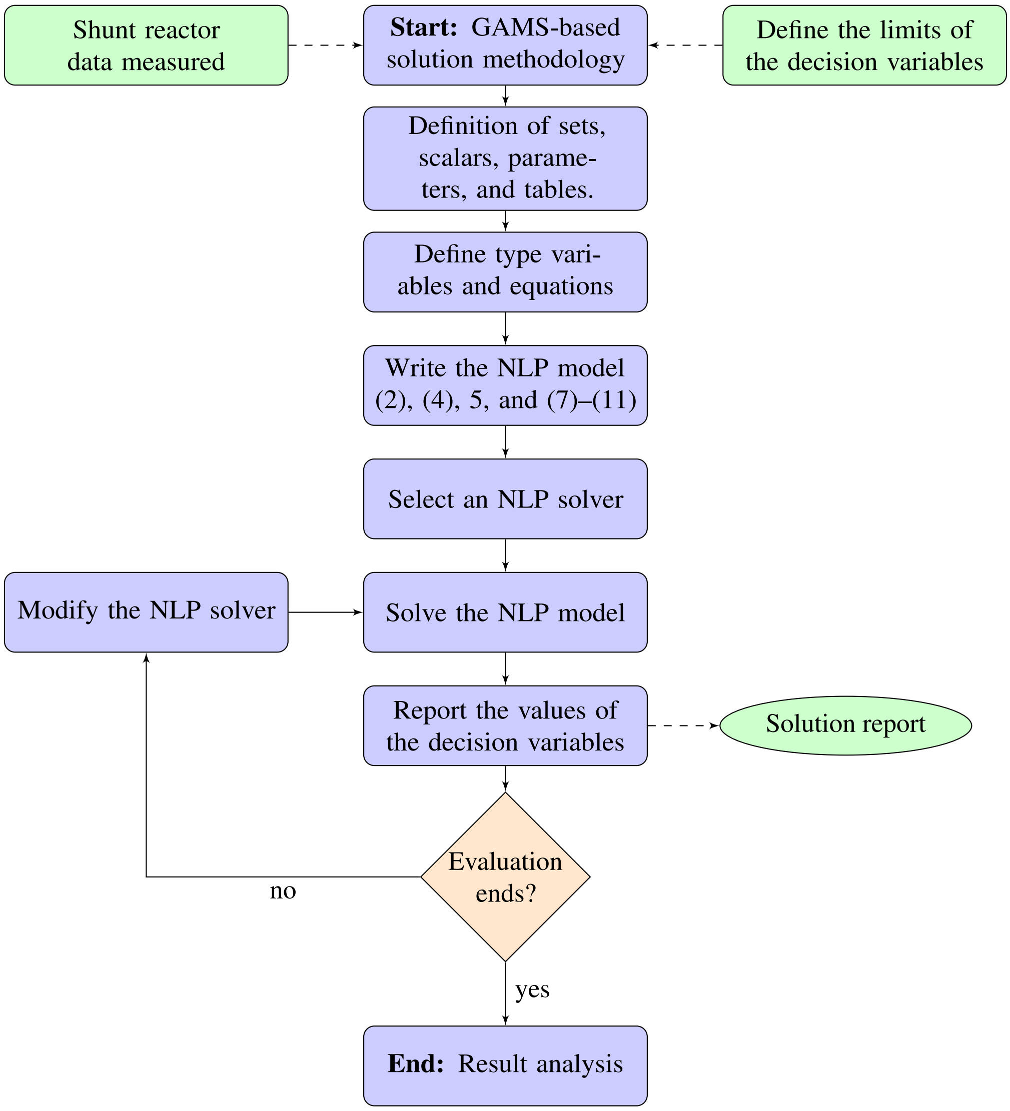

Figure 4: General steps for implementing a nonlinear programming model in the GAMS software

6.1 First reactor

Table 6 presents the numerical solutions reached with each one of the NLP solvers for the first test shunt reactor.

Tabla 6: Solutions reached with the GAMS solvers in the 13 kV, 2 Mvar reactor (all values in Ω)

	GAMS solvers		
Par.	**CONOPT**	**COUNNE**	**IPOPT**
R_s	0.4723	0.7207	0.7218
X_s	4.5000	3.3876	3.4825
R_M	4000	4868.1521	4860.5995
X_M	79.9811	81.0840	80.9891
Z_r	84.4746	84.4746	84.4746
MSE	2.5548×10^{-11}	2.5547×10^{-11}	2.5547×10^{-11}

Numerical results in Table 6 show that the problem of the parametric estimation in shunt reactors for MV applications has multiple optimal solutions, i.e., it is a multi-modal optimization problem since each one of the selected solvers founds the same objective function value with a combination of different series and parallel parameters; however, when the magnitude of the impedance (Z_r) is observed, all the solvers found precisely the same value for this parameter, i.e., 84.4746 Ω. In addition, if with voltages and currents listed in Table 2 it is obtained the average impedance based on measures, this takes the value of 84.4745 Ω, i.e., the same value reported by the different solvers in Table 6 which confirm that effectively all these solutions are optimal.

6.2 Second reactor

Table 7 presents the numerical solutions reached with each one of the NLP solvers for the second test shunt reactor.

The main characteristic of the results in Table 7 is that, as happened for the first reactor, all the GAMS solvers reach the same objective function value, i.e., 6.1093×10^{-12}, which confirms that the parameter estimation in single-phase re-

Tabla 7: Solutions reached with the GAMS solvers in the 25 kV, 6.75 Mvar reactor (all values in Ω)

	GAMS solvers		
Par.	**CONOPT**	**COUNNE**	**IPOPT**
R_s	0.4292	0.7159	1.0560
X_s	10	8.8134	6.1011
R_M	5000	6513.6306	10146.7505
X_M	82.5804	83.7582	86.4631
Z_r	92.5752	92.5752	92.5752
MSE	6.1093×10^{-12}	6.1093×10^{-12}	6.1093×10^{-12}

actor for MV applications is effectively a multi-modal optimization problem since multiple variable combinations provide the same equivalent impedance, i.e., 92.5752 Ω.

7 Conclusions

This research presented a nonlinear programming model to characterize the electrical circuit of shunt reactors in MV applications. The proposed NLP model minimizes the expected error between the measured and calculated variables using the mean square error as a performance indicator. The set of constraints included the equivalent impedance calculation in the terminals of the shunt reactor, the current flow on its terminals, and the active and reactive power behavior, respectively. The solution of the NLP model is reached by implementing it in the GAMS software with three different NLP solvers known as CONOPT, COUENNE, and IPOPT.

Numerical results in two shunt reactors showed that: (i) the objective function in both test shunt reactors was lower than 1×10^{-10}, which confirmed that the electrical measured and calculated variables have the same numerical performance; and (ii) the solutions reached by each solver differ on the particular parameters. However, their combination produces the same equivalent impedance (84.4746 Ω for the first reactor and 92.5752 Ω for the second reactor). The differences in the single parameters confirmed that, due to the solution space's nonlinearities, the parametric estimation in MV reactors is a multi-modal optimization problem with multiple optimal solutions with the same objective function value.

In future works, it will be possible to conduct the following works: (i) the application of combinatorial optimization methods to solve the proposed NLP model, and (ii) to extend the proposed NLP model to characterize electrical circuits in shunt reactors subject to saturation effects on the ferromagnetic core.

Appendix

Here, the GAMS code for the second reactor (25 kV, 6.75 Mvar) is presented in Algorithm 1.

Declaración de conflicto de interés: Los autores manifiestan no tener conflictos de interés.

```
SETS
k Set of measured data /k1*k11/;
SCALARS
Rsmin Minimum series resistance value /0.25/
Rsmax Maximum series resistance value /2/
Xsmin Minimum series reactance value /2/
Xsmax Maximum series reactance value /10/
RMmin Minimum parallel resistance value /5000/
RMmax Maximum parallel resistance value /15000/
XMmin Maximum parallel reactance value /50/
XMmax Maximum parallel reactance value /125/
TABLE Measures(k,*)
      Vrk    Irk     Prk        Qrk
k1    18750  202.538 73541.785  3796875.312
k2    20000  216.041 83674.209  4320000.355
k3    21250  229.543 94460.338  4876875.401
k4    22500  243.046 105900.171 5467500.449
k5    23750  256.548 117993.709 6091875.501
k6    25000  270.051 130740.952 6750000.555
k7    26250  283.553 144141.899 7441875.612
k8    27500  297.056 158196.552 8167500.671
k9    28750  310.558 172904.909 8926875.734
k10   30000  324.061 188266.970 9720000.799
k11   31250  337.563 204282.737 10546875.867;
VARIABLES
MSE Objective function
Zr  Equivalent impedance
Ir(k) Calculated current
Pr(k) Calculated active power
Qr(k) Calculated reactive power
VM(k) Calculated voltage in the parallel branch
Rs Series resistance
RM Parallel resistance
Xs Series reactance
XM Parallel reactance;
Rs.lo = Rsmin; Rs.up = Rsmax;
Xs.lo = Xsmin; Xs.up = Xsmax;
RM.lo = RMmin; RM.up = RMmax;
XM.lo = XMmin; XM.up = XMmax;
Zr.lo = Rsmin;
Equations
Obj Objective function equation
Pactive(k) Active power equation
Qreactive(k) Reactive power equation
Current(k) Input current equation
Impedance Input impedance
Vmag(k) Parallel voltage equation;
Obj.. MSE =E= (1/2)*sum(k,sqr(Measures(k,'Irk') - Ir(k))/sqr(Measures(k,'Irk')
        ))+
              (1/2)*sum(k,sqr(Measures(k,'Prk') - Pr(k))/sqr(Measures(k,'Prk')
                  ))+
              (1/2)*sum(k,sqr(Measures(k,'Qrk') - Qr(k))/sqr(Measures(k,'Qrk')
                  ));
Pactive(k).. Pr(k) =E= Rs*sqr(Ir(k)) + (1/RM)*sqr(VM(k));
Qreactive(k).. Qr(k) =E= Xs*sqr(Ir(k)) + (1/XM)*sqr(VM(k));
Impedance..  Zr =E= sqrt(sqr(Rs + RM*sqr(XM)/(sqr(RM) + sqr(XM))) +
                    sqr(Xs + XM*sqr(RM)/(sqr(RM) + sqr(XM))));
Current(k).. Ir(k) =E= Measures(k,'Vrk')/Zr;
Vmag(k).. VM(k) =E=  RM*XM*Measures(k,'Vrk')/(Zr*sqrt(sqr(RM) + sqr(XM)));
MODEL Estimator /ALL/;
Options decimals = 4;
SOLVE Estimator us NLP min MSE;
DISPLAY MSE.l, Rs.l, Xs.l, RM.l, XM.l, Zr.l, Pr.l, Qr.l ;
```

Algorithm 1: GAMS implementation of the parametrization model for parametric estimation for shunt reactors in MV applications (implementation of the second test reactor).

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