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Water stress in potato, corn and pea crops

El estrés hídrico de cultivos de papa, maíz y arveja

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ABSTRACT

The evapotranspiration of potato, corn and pea crops in the central region of Boyacá (Colombia) is analyzed under standard conditions and under hydric stress to determine water requirements and optimal planting calendars according to water balances. The study included the collection and purification of information that has a greater impact on these phenomena such as climatological variables, crop physiology and soil types; Mathematical models that describe evapotranspiration and water needs were designed and adjusted, which were implemented computationally through the simulation of all possible crops under study in the analysis period from 1967 to 2019. As results, critical periods were determined and optimal planting dates were established for each crop.

Keywords: evapotranspiration, precipitation, water needs, water balance.

RESUMEN

Se analiza la evapotranspiración de los cultivos de papa, maíz y alverja en la región central de Boyacá (Colombia) en condiciones estándar y bajo estrés hídrico para determinar requerimientos de agua y calendarios óptimos de siembra según los balances hídricos. El estudio comprendió la recolección y depuración de información que tiene mayor incidencia en estos fenómenos como las variables climatológicas, la fisiología de los cultivos y los tipos de suelo; se diseñaron y ajustaron modelos matemáticos que describen la evapotranspiración y las necesidades de agua, los cuales se implementaron computacionalmente a través de la simulación de todos los posibles cultivos objeto de estudio en el periodo de análisis de 1967 a 2019. Como resultados se determinaron los periodos críticos y se establecen las fechas de siembra óptimas para cada cultivo.

Palabras Clave: evapotranspiración, precipitación, necesidades de agua, balance hídrico.

1 INTRODUCTION

The climate is a complex system that in the central region of Boyacá presents a lot of variability, either due to excess or lack of rain and meteorological frosts, among other adverse phenomena that affect the evaporation and transpiration phenomena of plants, which determine the development of the crops [1].

The uncertainty of the behavior of water systems in the soil and plants impacts the agricultural production of the region, affected either by the lack or excess of water to satisfy the optimal conditions of the crops, generating shortages or overproduction, affecting the economy of the growers [2].

In response to this problem, this research aims to answer the following question: How to formulate, resolve, simulate and validate mathematical models that describe the dynamics of soil water of potato, corn and peas crops in the central region of Boyacá?

To answer this question, evaporation was related to transpiration that is integrated into the phenomenon called evapotranspiration, symbolized ET, a phenomenon that is affected by crop factors such as: type, variety and stages of development; and by the climatic variables of solar radiation, air temperature, atmospheric humidity and wind speed. [3, 4].

Subsequently, the water balance between the evapotranspiration of the crops under study and precipitation is analyzed through the simulation of all possible crops with information from the weather stations of the Institute of Hydrology, Meteorology and Environmental Studies (IDEAM) located in the region, the physical and physiological properties of the crops and the characteristics of the soil [5, 6, 7].

2 METHODOLOGY

The description and analysis of the water requirements of the simulated crops between 1967 and 2019 included the following phases [3, 5, 8]:

F-1: Calculate the daily evapotranspiration of a reference crop, noted as ET_o, from January 1, 1967 to February

20, 2020, equation (1).
For each of the potato, corn and pea crops:

- ***F-2:*** Determine the duration in days of each of the stages: initial, development, mid and final.
- ***F-3:*** Adjust reference parameters of each crop and the soil to the conditions of the region.
- ***F-4:*** Calculate the unique crop coefficient, noted as K_c, in each of its stages.
- ***F-5:*** Calculate daily evapotranspiration under standard conditions, equation: $ET_c = K_c \cdot ET_o$.
- ***F-6:*** Calculate the dual coefficient of the crop, as the sum of the basal and evaporation coefficient, $K_c = K_{cb} + K_e$, the basal to determine the water loss through transpiration $K_{cb} \cdot ET_o$ and the evaporation K_e to determine the water loss due to evaporation $K_e \cdot ET_c$.
- ***F-7:*** Calculate the daily evapotranspiration of each crop ET_c under standard conditions through the dual coefficient, $K_c = (K_{cb} + K_e) \cdot ET_0$.
- ***F-8:*** Calculate the daily evapotranspiration of each crop under non-standard conditions through the hydric stress coefficient, $ET_{c\ aj} = (K_s \cdot K_{cb} + K_e) \cdot ET_o$.
- ***F-9:*** Calculate the daily and accumulated water balance.
- ***F-10:*** Simulate all possible crops and analyze the behavior of hydric stress during the study period.
- ***F-11:*** Formulate a model to describe crop hydric stress and optimal planting schedules.

3 RESULTS

3.1 *The ET of the reference crop, ET_o.*

This concept corresponds to the evapotranspiration of a hypothetical grass crop with specific characteristics and means the loss of water from a standard cultivated area. It also expresses the evaporative force of the atmosphere in a specific locality and time of year, affected only by climatic parameters.

For its daily calculation, when meteorological data on air temperature, atmospheric humidity, wind speed and radiation were available, the Penman-Monteith model was applied, equation (1), [5].

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma\frac{900}{T+273}u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \quad (1)$$

where:

Symbol	Meaning	Unit
ET_o	Reference evapotranspiration	$mm\,day^{-1}$
R_n	Net radiation on the crop surface	$MJ\,m^{-2}\,day^{-1}$
R_a	Extraterrestrial radiation	$mm\,day^{-1}$
G	Soil heat flow	$MJ\,m^{-2}\,day^{-1}$
T	Average air temperature	C
u_2	Wind speed at 2 m height	$m\,s^{-1}$
e_s	Saturation vapor pressure	kPa
e_a	Vapor real pressure	kPa
$e_s - e_a$	Vapor pressure deficit	kPa
Δ	Vapor pressure curve slope	$kPa\,C^{-1}$
γ	Psychrometric constant	$kPa\,C^{-1}$

However, when data corresponding to the model were not available (1) the alternative Hargreaves model was applied (2) with information on solar radiation R_a, temperature, and in cases where these data were not available, they were interpolated by adjustment with polynomials [4].

$$ET_o = 0.0023(T_{media} + 17.8)(T_{máx} - T_{min})^{0.5}R_a \quad (2)$$

As a result of the computational implementation of the models, the ET_o was found daily in the 53 years of analysis, from 1967 to 2019, with climatological data from the station of the Pedagogical and Technological University (Uptc) located at $5^o32'25"$ N y $73^o21'41"W$ (Grw) and at an elevation of 2690 msm.

In this respect, the ET_o value varies between 1.12 and 4.99 mm/day. The maximum accumulated value of ET_o was in the year 1967 of 1182.62 $mm/year$, which means that in that year there was greater loss of water from the reference crop by evaporation and transpiration, while the minimum is 1006.29 $mm/year$ in 1984, which means otherwise.

Figure1 represents the behavior of the ET_o for the year 1979 in which the optimal conditions for potato cultivation occurred.

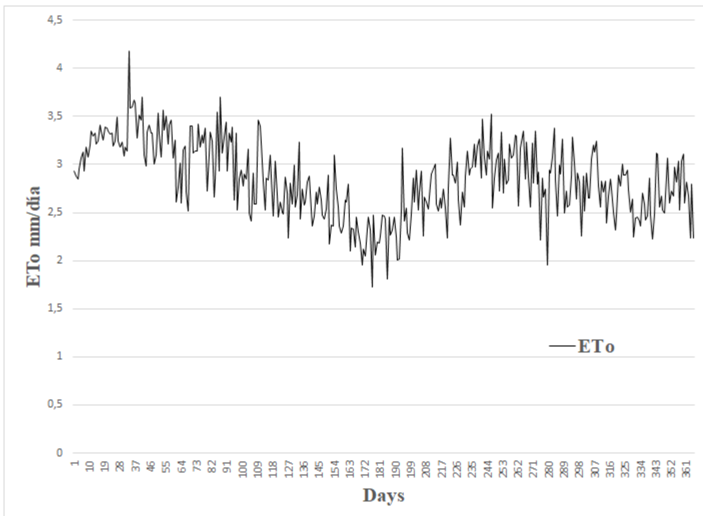

Fig. 1. Reference evapotranspiration ET_o for the year 1979.

With respect to precipitation PP (mm/day), this varies in the range $[0, 66.9]$, with maximum values presented on 8 October 1995 at 66.9 mm/day; on 21 April 2012 at 56.4 mm/day and on 29 September 1980 at 45.7 mm.

3.2 *The ET of the crop under standard conditions, ET_c*

The unique crop coefficient K_c integrates the physical and physiological variational characteristics that distinguish the crops under analysis [5]. The reference parameters for the crops required for their calculation are described in Table 2 and an example of the function of the unique crop coefficient adjusted the values to the conditions of the region is presented in Figure 2, [3].

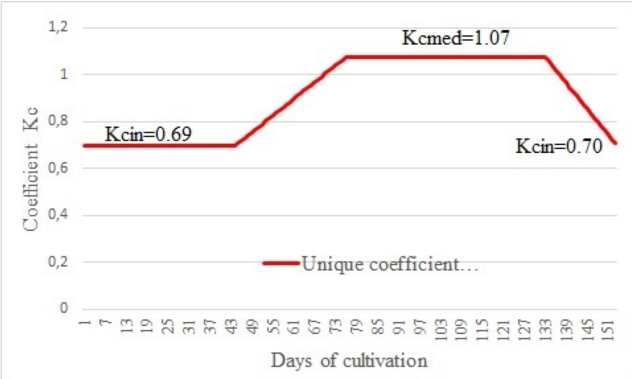

Fig. 2. K_c of potato, crop 21/03/1979-20/08/1979

Regarding the evapotranspiration of the crop under standard conditions (ET_c), it was calculated with the equation (3), which refers to the evaporative demand of the atmosphere on crops growing in large areas under optimal soil water conditions, with appropriate management and environmental characteristics and reaching potential production under the given climatic conditions and according to the stages of development listed in Table 2, see [9].

$$ET_c = K_c \cdot ET_o \tag{3}$$

Figure 3 shows the ET_c of the potato crop under standard conditions, ranging from 1.65 to 3.40 mm/day in each of the stages of cultivation, it is inferred that in the initial stage the ET_c is less, where evaporation has more influence on the phenomenon, while in the middle stages the one that has more impact on the ET_c is transpiration.

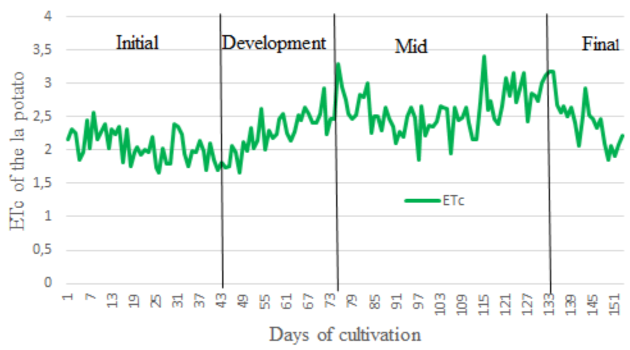

Fig. 3. ET_c of the la potato, crop 21/03/1979-20/08/1979

In this sense, the water need of a crop is defined as the amount of water required to compensate for the loss due to the ET_c of the crop and the water balance represents the difference between the water requirement and the effective precipitation. PP (mm/day, [6], according to equation (4).

$$NA = ET_c - PP \tag{4}$$

For the potato, 19265 possible crops were simulated, the first from January 1 to June 2, 1967 and the crop number 19265 from September 30, 2019 to February 29, 2020, according to available climatological data. Table 1 shows the results of water needs of the crops under standard conditions.

From this Table in the second row, the values of the ET_c are presented, the minimum is 295.70 mm/day that was recorded in the crop from February 11 to July 13, 1977 and the maximum of 490.34 mm/day that was recorded in the crop from June 22 to November 21, 1977.

From the accumulated value of water needs, equation 4, infer that in the crop from 16/02/2011 to 18/07/2011 the loss of water by ET_c was higher than the compensation by precipitation; while the crop from 26/10/2015 to 26/03/2016 presented higher PP to compensate for the loss of water by ET_c.

The value closest to zero of the water needs, $NA = 0.02493$, was estimated in the crop from 21/03/1979 to 20/08/1979, meaning that there was a better balance between water loss per ET_c and rain compensation; Therefore, in this period the optimal weather conditions for cultivation under standard conditions were presented. To establish water losses from the plant in a discriminated manner both by evaporation and transpiration, the dual coefficient defined as the sum of the basal coefficient of the plant K_{cb} and the evaporation coefficient K_e is required, [10], equation (5).

Table 1. Critical values for the simulation period 1967 to 2020

(1)	(2)	(3)	(4)	(5)
ETc	295.70	11/02/19977	490.34	22/06/1967
		13/07/1977		21/11/1967
PP	62.20	24/10/2015	745.20	16/02/2011
		24/03/2016		18/07/2011
NA	-412.30	16/02/2011	417.83	26/10/2015
		18/07/2011		26/03/2016
(6)	-0.04	2/04/1971	0.02	21/03/1979
		1/11/1971		20/08/1979

(1) : Accumulated crops.
(2) : Minimum mm/day.
(3) : Crop duration dates, accumulated minimums.
(4) : Maximum mm/day.
(5) : Crop duration dates, accumulated maximums.
(6) : $NA = ET_c - PP \approx 0$.

$$K_c = (K_e + K_{cb}) \tag{5}$$

With this coefficient it is possible to calculate the water loss by evaporation, $ET_o \cdot K_e$ and by transpiration, $ET_o \cdot K_{cb}$, [11], equation (6).

$$ET_c = (K_e + K_{cb}) \cdot ET_o \tag{6}$$

With respect to the evaporation component, the coefficient K_e is calculated by the equation (7).

$$K_e = \min\left(K_r(K_{c\ máx} - k_{cb,i}), f_{ew,i} \cdot K_{c\ máx}\right) \tag{7}$$

The evaporation reduction coefficient K_r depends on the amount of water depleted (evaporated) from the surface layer of the soil and is calculated with the equation (8).

$$K_r = \frac{AET - D_{e,i-1}}{AET - AFE},\ D_{e,i-1} > AFE \tag{8}$$

In equation (8) AET is the total evaporable water in (mm), equation (9), which corresponds to the sheet of water that can be evaporated from the soil when the layer evaporative has been completely moistened; AFE is the easily evaporable water that corresponds to the water layer at the end of stage 1 of the crop, measured in mm; $D_{e,i-1}$ is the accumulated evaporation (depletion) layer in the surface layer of the soil at the end of day $i-1$ (previous day) measured in mm. The coefficient $K_r = 1$ when $D_{e,i-1} \leq AFE$.

$$AET = 1000\,(\theta_{FC} - 0.5\theta_{WP})\,Z_e \tag{9}$$

In the equation (9) θ_{FC} is the moisture content in the soil at field capacity, θ_{WP} is the moisture content at the permanent wilting point, is taken as a reference for the type of loam soil in the region $\theta_{FC} = 0.29$ and $\theta_{WP} = 0.14$, $AFE = 8mm$; Z_e is depth of the evaporative layer of the soil that is subject to drying and varies in the range of $[0.1, 0.15]$ m; The maximum single coefficient value, $K_{c\ máx}$ is calculated with the equation (10), where h is the average maximum height of the plant at its stages, HR_{min} (%) is the minimum relative humidity [5].

$$K_{c\ máx} = \max\{v1, v2\} \tag{10}$$

Where,

$$\begin{aligned} v1 &= 1.2 + [0.04\,(u_2 - 2) - 0.004\,(HR_{mín} - 45)]\left(\frac{h}{3}\right)^{0.3}. \\ v2 &= K_{cb} + 0.05. \end{aligned}$$

Furthermore, in equation (11): $1 - fc$ is the average exposed fraction of the soil that is not covered by vegetation and varies in the range $[0.01, 1]$ m, f_c is given by the equation (12); f_w is the average fraction of the surface wetted by irrigation or rain.

$$f_{ew} = \min(1 - f_c, f_w) \tag{11}$$

In equation (12): $K_{c\ min}$ is the minimum value of K_c for an uncovered and dry soil, while $K_{c\ máx}$ is the maximum value of K_c for a soil after wetting.

$$f_c = \left(\frac{K_{cb} - K_{c\ mín}}{K_{c\ máx} - K_{c\ mín}}\right)^{(1+0.5h)} \tag{12}$$

In equation (13): i represents the ith day; $D_{e,i}$ is the accumulated depletion sheet at the end of the day; PP_i, is the precipitation; RO_i is the water runoff; I_i is the irrigation sheet; $T_{ew,i}$ is the transpiration sheet that occurs in the exposed and wetted fraction of the surface layer; $DP_{e,i}$ are the deep percolation losses that occur from the surface layer when the moisture content in the soil exceeds the field capacity.

$$D_{e,i} = D_{e,i-1} - (PP_i - RO_i) - \frac{I_i}{f_w} + \frac{E_i}{f_{ew}} + T_{ew,i} + DP_{e,i} \tag{13}$$

Regarding the transpiration component, the basal plant coefficient K_{cb}, for some specific crops they are recommended in [5]. However, these must be adjusted when the relative humidity is different from 45% or the wind speed is greater or less than 2 m/sec according to equation (14), where $K_{cb\ ref}$ are the reference values of $K_{cb\ med}$ or $K_{cb\ end}$.

$$K_{cb} = K_{cb\ ref} + [0.04(u_2 - 2) - 0.004(HR_{min} - 45)]\left(\frac{h}{3}\right)^{0.3} \tag{14}$$

3.3 The ET of the crop under hydric stress, $ET_{C\ aj}$

This phenomenon occurs when in the ET the crop field conditions differ from the standard ones, caused by low soil fertility, saline toxicity, flooded soils, pests, diseases, hard or impenetrable horizons in the root zone, which can generate poor plant growth and a reduction in ET and therefore corrections to this phenomenon are required.

When the potential energy of soil water falls below a certain threshold value, the crop is said to be stressed. The effects of hydric stress are incorporated by multiplying the basal coefficient of the crop by the hydric stress coefficient, K_s, equation (15), [8].

$$ET_{C\ aj} = (K_s \cdot K_{cb} + K_e) \cdot ET_0 \tag{15}$$

In case limitations occur due to insufficient water in the soil, $K_s < 1$, otherwise $K_s = 1$. The coefficient K_s describes the effect of hydric stress on crop transpiration and is calculated using the equation (16), where: ADT is the total available water in the root zone of the plant, equation (17) measured in mm; $D_{r,i}$ is the moisture depletion in the root zone of the plant, equation (20) measured in mm; AFA is the easily usable water extractable from the root zone of the soil equation (18).

$$K_s = \frac{ADT - D_{r,i}}{ADT - AFA} \tag{16}$$

In equation (17): θ_{FC} is the water content at field capacity; θ_{WP} is the moisture content at the permanent wilting point; Z_r is the depth of the plant's roots measured in meters.

$$ADT = 1000(\theta_{FC} - \theta_{WP}) \cdot Z_r \tag{17}$$

Regarding AFA equation (18), ρ is the average total fraction of available water ADT that can be exhausted from the root zone before hydric stress occurs (reduction of ET) and takes values in the range $[0, 1]$.

$$AFA = \rho \cdot ADT \tag{18}$$

The value of ρ is calculated using the equation (19) where ρ_{ref} is a reference value for each crop.

$$\rho = \rho_{ref} + 0.04(5 - ET_c) \tag{19}$$

The daily water balance (i) of the root zone of the plant is calculated by the equation (20) where: $D_{r,i-1}$ is the depletion in the root zone of the soil at the end of day $i-1$, PP_i is the precipitation, RO_i is the surface runoff; I_i is the irrigation layer that infiltrates the soil; $ET_{c,i}$ is the evapotranspiration of the crop; DP_i, are the deep percolation losses.

$$D_{r,i} = D_{r,i-1} - (PP_i - RO_i) - I_i - CR_i + ET_{C,i} + DP_i \tag{20}$$

In equation (20) initial depletion is estimated through measurements of soil moisture content given by equation (21), where θ_{i-1} corresponds to the average moisture content in the depth of the root zone of the plant.

$$D_{r,i-1} = 1000(\theta_{FC} - \theta_{i-1}) \cdot Z_r \tag{21}$$

To analyze the ET_c under hydric stress, with the ET_c estimated through the dual coefficient, the positive difference between easily usable water and depletion (if $D_{r,i} \geq AFA$, then $EH = Dr,i - AFA$, otherwise $EH = 0$); and the accumulated depletion in the crop was calculated numerically with the integral of the hydric stress function, equation (22), with $t_1 = 1,\ t_n = 153$ for the case of potato [12].

$$EHA = \int_{t_1}^{t_n} EH(t)\,dt = \frac{1}{2}\left(EH(1) + \sum_{i=2}^{t_n-1} + EH(t_n)\right) \quad (22)$$

Result of the computational implementation of the models described above and simulated the 19265 possible crops, the crop with minimum EHA was 97.45 mm, recorded from 31 July to 30 December 1988. In Figure 4, the graphs of the evaporation coefficients K_e varying in the interval $[0.026, 0.894]$ and the basal coefficient K_{cb} are represented with the values in the initial stages, mid and end.

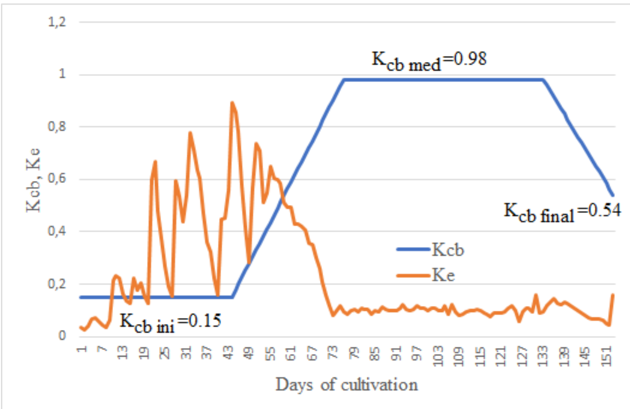

Fig. 4. K_{cb} and K_e crop: $31/07/1988 - 30/12/1988$

Figure 5 represents the minimum ET_c of the crop, calculated as the product of the dual coefficient (the sum of the basal and vaporization coefficients) with the reference evapotranspiration. The ET_c varies between 0.477 and 3.723 mm.

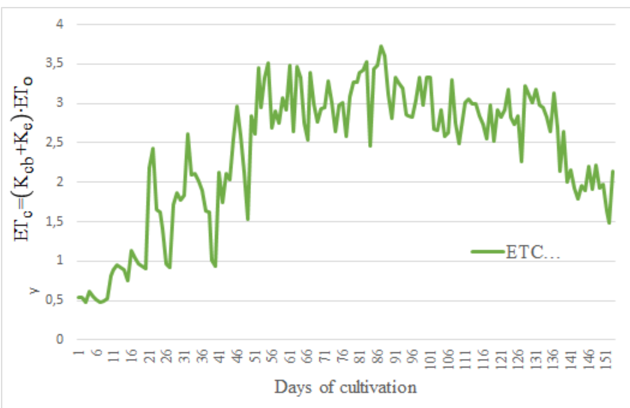

Fig. 5. $ET_c = (k_{cb} + K_e) \cdot ET_0$, crop $31/07 - 30/12/1988$

In Figure 6, the depletion of the evaporative layer of the crop is represented, which varies between 1.94 and 21.73 mm, precipitation, easily evaporable water $AFE = 9$ mm, total evaporable water $AET = 22$ mm, values taken as a reference for the typical moisture characteristics of a loam type soil [5].

It can be seen in the graph that precipitation has the effect of reducing depletion in the evaporative layer.

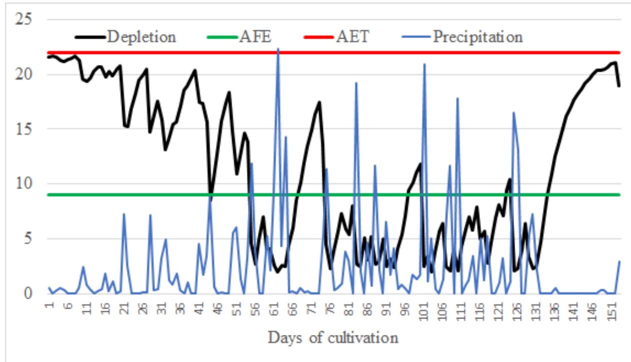

Fig. 6. $D_{e,i}$, crop $31/07/1988 - 30/12/1988$

In Figure 7, the depletion of the root zone is represented for the crop that showed minimal water stress. It is observed that if the depletion curve is below the easily usable water curve AFA the crop presents hydric stress, if it exceeds the total available water threshold ADT, the plant wilts. Furthermore, the presence of precipitation causes depletion to decrease. Hydric stress is calculated as the area between the depletion curves and the easily usable water, as long as the depletion curve is above the AFA curve.

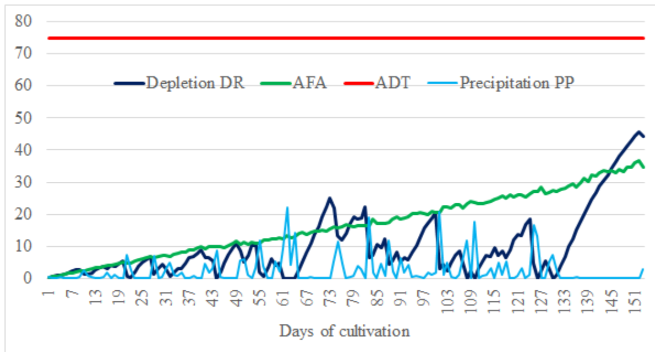

Fig. 7. $D_{r,i}$, crop $31/07/1988 - 30/12/1988$

On the other hand, potato culture, of the simulated with maximum hydric stress in the period of analysis, was presented from 11 May to 10 October 2009 with a value of $SH = 4963.46$ mm. par The year with the minimum accumulated HS of all possible crops was 2010 with 389467.99 mm and the year with the maximum accumulated HS was 2015 with 1174902.26 mm.

Finally, to determine the behavior of HS in all crops of a year, the following results are summarized in Table 3:

In row 1 the possible 365 crops for each year of analysis are indicated; in the second row the sowing date; For example, for the first crop the sowing is on January 1st, for the second crop it is on January 2nd and successively until the 365 crop with sowing date on December 31.

In row 3, samples of the accumulated hydric stress results are presented for the specific year of 1988, and for example the crop with sowing date on February 1 of 1988 and final date on $(2-VII-1988)$) the accumulated hydric stress was 1158.33 *mm* and similarly for the other crops of that year, all the crops from the other years were processed in the simulations.

In row 4, the minimum accumulated values of hydric stress among the crops with sowing date on each of the days of the year are presented, for example of the possible 53 years of crops (1967 to 2019) with date of planting on January 1 of each of these years, the crop that presented the minimum hydric stress was 464.50 *mm*, which corresponded to the sowing of 1 of January 2011.

Similarly, in row 5, the maximum accumulated estimated values of hydric stress are presented; For example, the crop that presented maximum hydric stress of those planted on 1 of January was 3646.29 *mm*, which corresponded to 1 January 1970.

In row 6 the average values for these crops are presented, for example, the crops sown on the first of January of each year, the average hydric stress is 1929.74 *mm*.

In Figure 8, the data from Table 3 is graphically represented. In this graph, the abscissa axis corresponds to the days of year, the sowing dates of each crop, and on the ordinate axis the hydric stress values for the year of 1988, the minimum, maximum and average values corresponding to rows 3 to 6 of Table 3.

From this Figure it is inferred that the band limited by the minimum and maximum hydric stress, where all the 53 graphs of the accumulated hydric stress that were simulated on each of the planting dates and the trend are found, *HS* average, which supports some estimates on the recommended planting calendars for potato cultivation in the study area.

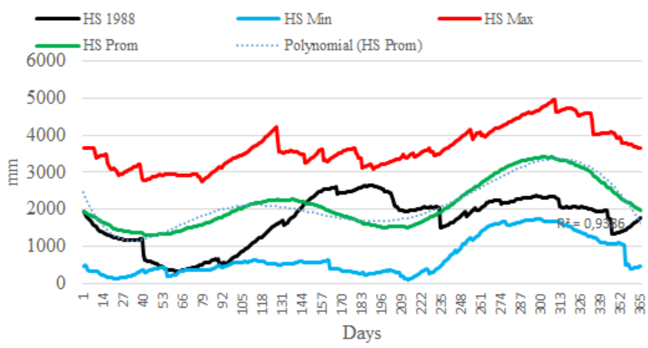

Fig. 8. History of hydric stress in potato crops

From Table 3 and Figure 8, with the data of the averages of hydric stress accumulated, is found that the polynomial that best represents them in the range $[1,365]$, by regression methods polynomial [13], it is of order 30 equation (23).

$$P_{30}(x) = 2057.90 - 103.44x + 25.63x^2 + \cdots - 5\times10^{-64}x^{30} \quad (23)$$

In Figure 9, the days of the year are represented on the abscissa axis, the averages of hydric stress and the polynomial graph are represented on the ordinates. The analysis is located in the interval $[1,365]$ which justifies the following statements: The real roots of $P''_{30}(x)=0$, inflection points, are: $x_0=5$, $x_1=12.7$, $x_2=95.2$, $x_3=164.9$, $x_4=247.3$, $x_5=95.2$, $x_6=341.9$; Therefore, the graph is convex in the intervals, $[12.7,95.2]\approx[13,9]$ which corresponds to the period from 13 in January to 5 in April and $[164.9,247.3]\approx[165,247]$, period from 14 in June to 4 in September; This means that in these periods they are recommended for the planting of the potato crop brown variety pastuses in this region.

Since the minimum or maximum values reached by the polynomial are found in the roots of the equation $P'_{30}(x)=0$. The minimum values are found at $x_7=47.3\approx47$, $x_8=30.5\approx130$ corresponding to 16 in February and 8 in July, this means that sowings made near these dates will provide optimal conditions for cultivation. These theoretical statements coincide with the empirical experiences of farmers for sowing crops at the beginning of the year (large year harvest) and mid-year (Saint Peter); which is validated with research published in [14]. This means that sowings made near these dates will provide optimal conditions for cultivation.

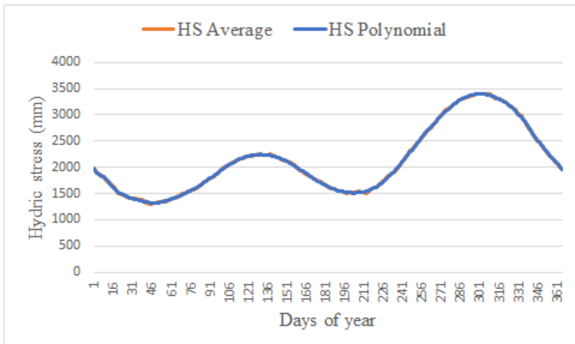

Fig. 9. Hydric stress curves for potato crops

On the other hand, in the other intervals, $y=P_{30}(x)$ is concave in the intervals $[96,164]$ and $[248,365]$, periods that are not recommended for sowing the crop, because high hydric stress will occur; In these intervals, the critical days where the maximums are reached are 130 and 307 of the year, which correspond to 10 in May and 27 in October, non-optimal dates for planting this crop.

Similarly for the pea crop, according to the data in Table 4, in Figure 10, the graphs of the accumulated hydric stress of the 365 crops simulated in the 53 are presented years of analysis, the corresponding minimums, maximums and averages

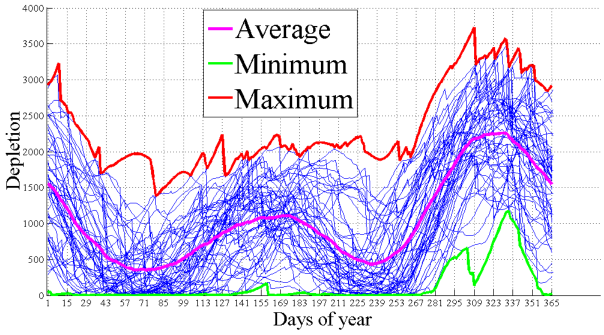

Fig. 10. Hydric stress curves for pea cultivation

In Figure 11, the maximum and minimum values of water levels are represented, the following critical values of pea crops were found

Minimum HS: 0 mm from $07/09/2010$ to $25/12/2010$.

Maximum HS: 3727 mm from $5/11/2009$ to $22/02/2010$.

Year with minimum HS accumulated: 2010 with 181449.15 mm.

Year with maximum HS accumulated: 2015 with 676459.07 mm.

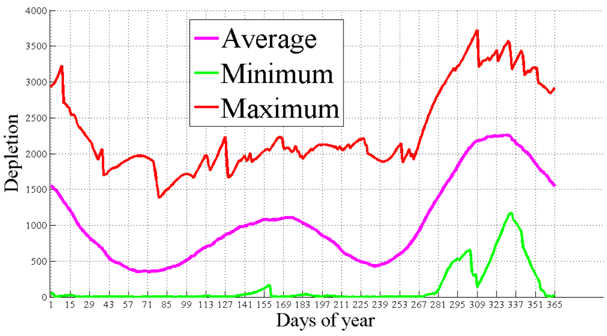

Fig. 11. Pea hydric stress curves

Figure 12 represents the trend of the accumulated stress curves and the polynomial regression, where it is found that the optimal planting periods are for the first harvest from 5 February to 21 April and the second between from 21 in July to 7 in October, optimal sowing dates, 11 in March and 24 in August; calendars that agree with those defined in [15], [16].

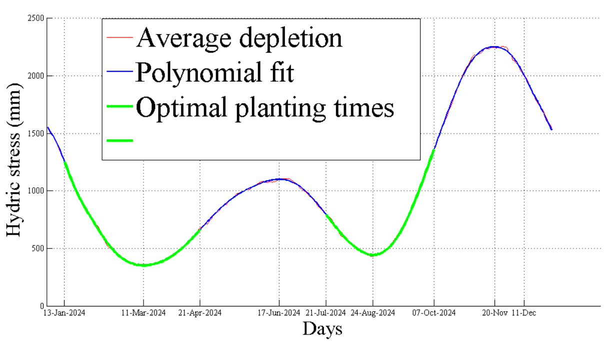

Fig. 12. Optimal pea sowing periods

Regarding the cultivation of corn in Figure 13, the maximum and minimum values of water levels are represented, the following critical values were found.

Minimum HS: 113.02 mm for 20/03/1973 to 13/01/1974

Maximum HS: 16153 mm for 6/06/1967 to 31/03/1968

Year with minimum HS accumulated: 2010 with 371497.11 mm

Year with maximum HS accumulated: 2015 with 4719153.68 mm

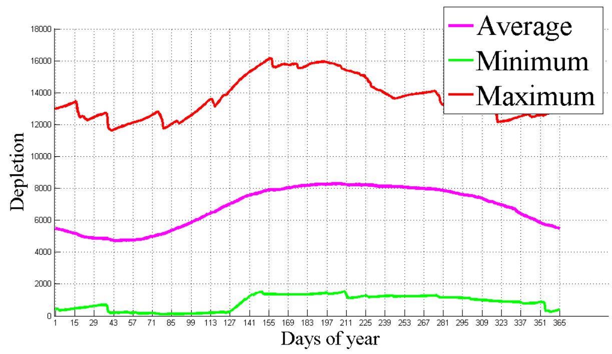

Fig. 13. Corn hydric stress curves

Figure 14 represents the trend of the accumulated stress curves and the polynomial regression, where it is found that the optimal period for the single crop in the year is from 4 February to 30 April and the date Optimum sowing time is 20 in February, which coincides with the studies of [15].

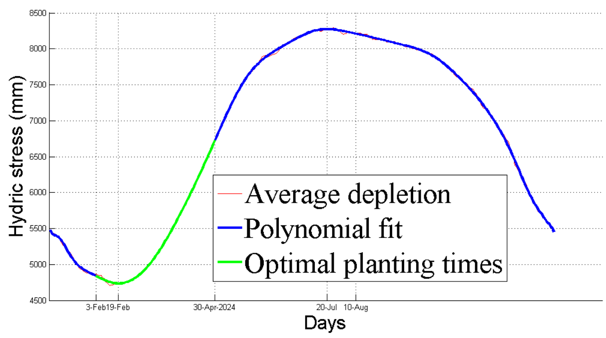

Fig. 14. Optimal Corn Planting Periods

4 CONCLUSIONS.

Regarding the water requirements of potato cultivation: With reference to the water needs $NA = ET_c - PP$, the lowest values occurred in the sowing crops in the months of December of 2010 to March of 2011, times with greater precipitation than ET_C, which coincides with the girl phenomenon; The values closest to 0, a tendency towards a balance between the PP and the ET_C, correspond to the years 1971 and 1979 in sowing dates in the months of January to March and in the years 2008 and 2009 with sowing in the months of July and August, the second harvest of the year, which coincides with the sowing calendars determined empirically by farmers; while the maximum values of NA were presented in the

Table 2. Crop parameters

		Cultivation stages days				Unique K_c reference			K_{cb} reference			Height	Depth	Fract.
Crop	Variety	Ini.	Deve.	Med.	End	Inic.	Med.	End	Ini.	Med.	End.	Máx.	Root	Deple.
Potato	Brown Pastusa	44	32	57	20	0.5	1.15	0.6	0.5	1.15	0.6	0.6	0.5	0.35
Corn	Yellow	20	136	90	54	0.35	1.2	0.54	0.15	1.15	0.15	2	1.35	0.55
Pea	Saint Elizabeth	35	25	30	20	0.5	1.15	0.3	0.15	1.15	0.15	0.5	0.8	0.4

Table 3. Potato hydric stress values in the analysis period, 1967 to 2019

No	**1**	**2**	···	**31**	**32**	**33**	···	**363**	**364**	**365**
Sowing date	1-I	2-I	···	31-I	1-II	2-II	···	29-XII	30-XII	31-XII
EH 1988 (mm)	1919.82	1870.02	···	1158.95	1158.33	1159.04	···	1697.90	1734.62	1753.09
EH Min (mm)	464.50	476.37	···	188.95	201.90	218.36	···	431.55	441.96	453.04
EH Max (mm)	3646.29	3643.17	···	3066.85	3094,3	3122,5	···	3665.46	3658.03	3650.75
EH Prom (mm)	1929.74	1909.03	···	1382.77	1380.79	1379.35	···	2010.06	1990.56	1964.00

Table 4. Pea hydric stress values in the analysis period, 1967 to 2019

No	**1**	**2**	···	**31**	**32**	**33**	···	**363**	**364**	**365**
Sowing date	1-I	2-I	···	31-I	1-II	2-II	···	29-XII	30-XII	31-XII
EH 2010 (mm)	1670.75	1621.69	···	630.53	621.71	613.92	···	5.77	11.18	24.14
EH Min (mm)	39.96	56.49	···	0	0.17	0.34	···	5.77	11.18	24.14
EH Max (mm)	2934.88	2956.27	···	2091.65	2050.06	2008.88	···	2866.59	2890.35	2912.95
EH Prom (mm)	1552.15	1534.05	···	789.18	773.18	758.32	···	1591.23	1574.04	1549.09

months of September to November of 2015 related to the presence of the El Niño phenomenon and in periods where the ET is greater than the precipitation PP, which coincides with periods little or no precipitation and adverse phenomena such as frost.

The previous conclusions are reaffirmed with the analysis of hydric stress, the year with minimum EH accumulated in the crop was 2010 with 389467.99 mm and the year with maximum EH accumulated was 2015 with 1174902.26 mm; Furthermore, the hydric stress range is $[97.45, 4963]$ mm.

The crop from 31 in July to 30 in December 1988 presented the minimum EH of 97.45 mm and due to the weather conditions recorded in this period, they are considered optimal for the crop and serve as a reference in technical management. through systems such as greenhouse, drip irrigation, sprinkler irrigation, among others.

Regarding the water requirements of pea cultivation: Two harvests are established per year, the first with sowing in the months of March and April, with recommendation in the middle of march; the second in the months of August and September, with recommendation in the last two weeks of August. The hydric stress range for this crop is $[0.3727]$ mm. The year with minimum hydric stress accumulated in the crops was in 2010, the beginning of the El Niño phenomenon and the maximum in the year 2015.

Regarding the water requirements of corn: It is a crop that has an average duration of 10 months, therefore only a one harvest is established in the year, the sowing calendar is from February to April with a recommendation in the last two weeks of February; The hydric stress range of this crop is $[113.02, 16153]$ mm.

Regarding the three crops under study, it is inferred that the pea is the crop that presents the least hydric stress, followed by the potato and finally corn.

In relation to the climatic conditions there are high variations in the study period, which affect the difficulty to determine with some precision their behavior that impact on the crops. However, according to mathematical models developed and validated together with the methods of approximation it was possible to identify the times with favorable climatic conditions that corroborate the projection of planting calendars.

The study can be applied in other regions where meteorological information is available for the same or other crop varieties, as well as for other crops in the region, generating localized information essential in making decisions that improve techniques and production of crops [17].

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