Concepts of probability: a study with Colombian university students

Conceptos de Probabilidad: un estudio con estudiantes Colombianos universitarios

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ABSTRACT

Probability is a complex concept to understand that has attracted the interest of many researchers. This study aimed to explore the perception and obstacles present in students taking probability and statistics courses for the first time at a Colombian university. The methodology included quantitative research elements and a descriptive-exploratory design. A deterministic sample of 313 students was selected. The diagnostic test showed that 24.92% of the participants associate probability with gambling, 29.39% are unaware of its use in their professional career, and 40.58% in their daily lives. The results indicated that its teaching has had a deterministic character with the presence of epistemological, didactic, and cognitive obstacles. It is concluded that the concept of probability is related to the knowledge acquired in their pre-university education and beliefs, requiring the implementation of didactic strategies to improve their levels of understanding.

Keywords: Probability, obstacles, first-year university students.

RESUMEN

La probabilidad es un concepto complejo de comprender que ha despertado interés en diversos investigadores. Este trabajo tuvo como objetivo, explorar la percepción y los obstáculos presentes en estudiantes que cursaban por primera vez asignaturas relacionadas con probabilidad y estadística en una universidad colombiana. La metodología incluyó elementos de investigación cuantitativa y un diseño descriptivo-exploratorio. Se seleccionó una muestra determinista de 313 estudiantes. El diagnóstico evidenció que un 24,92% de los participantes asocian la probabilidad con los juegos de azar, el 29,39% desconoce su uso en su carrera profesional y un 40,58% en su vida diaria. Los resultados indicaron que su enseñanza ha tenido un carácter determinista con presencia de obstáculos epistemológicos, didácticos y cognitivos. Se concluye que el concepto de probabilidad es relacionado con el conocimiento adquirido en su educación pre universitaria y en sus creencias, requiriéndose la implementación de estrategias didácticas para mejorar sus niveles de comprensión.

Palabras Clave: Concepciones de probabilidad, obstáculos, estudiantes universitarios.

1 INTRODUCTION

The study of the understanding of the concept of probability and stochastics in general has attracted the interest of many researchers, some of whom focus on its didactics: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. These researchers have converged on the investigation of the presence of obstacles in the teaching-learning process of the conceptions or meanings of probability. This problem has also been considered by some international organizations, such as [1] the National Council of Teachers of Mathematics, The National Council of Supervisors of Mathematics, International Conference on Teaching Statistics, International Asociation for Statistical Education, American Educational Research Association, Royal Statistical Society, Sociedad Estadística Japonesa, Sociedad Española de Investigación en Educación Matemática, Statistical Education Research Group, The International Commission Mathematical Instruction, and the Red Latinoamericana de Investigación en Educación Estadística.

However, the difficulties of learning and teaching probability persist, despite the rapid scientific advances that are taking place alongside the use of probability. This is evident in the existence

of a "knowledge gap in terms of statistics and probability, due to the fact that, for years, it was not considered a necessity" [16]. Furthermore, these topics have been invisible or very poorly addressed in the mathematics curriculum in classroom sessions. This is even though the range of applications of probability is very broad and probabilistic language should be part of everyday life [18]. Thus, it is necessary to recognize that "probabilistic knowledge has been useful throughout history to answer questions related to chance or to study situations in which prediction is required" [19].

In the same way, probability "is limited to the superficial teaching of the classical meaning, neglecting the different meanings it has. The perception by teachers who grew up without understanding probabilistic concepts limits the teaching and learning process of necessary knowledge" [20]. This statement justifies analyzing the perception of the concept of probability held by 313 students from a Colombian university who took for the first-time courses related to probability or statistics in various academic programs. The study also aims to investigate obstacles, impediments, or anything that opposes learning. This is because "from the role of consumers of data, producers, or communicators of data, each student must achieve statistical and probabilistic literacy" [21], to make inferences and take decisions in uncertain situations in their academic, professional, or research lives.

In fact, the "Theory of Obstacles in Teaching and Learning and their Ways of Coping"has been developed by different authors, including [22]. Therefore, it is necessary to identify them and try to overcome them. However, this task is not easy because "in particular teaching contexts, there are impediments whose causes are found in the student profile, the teacher profile and performance, and, in some cases, in the object of knowledge itself" [23]. Therefore, it is necessary to recognize whether students presented ontogenetic, conceptual, didactic, or epistemological obstacles.

According to the difficulties presented above, from the reports in the literature "on the epistemology of probability, cognitive complexity, errors, and difficulties in probabilistic reasoning, it is held that young people and adults have informal ideas and prejudices in probabilistic situations, in which chance intervenes" [24], the deficiencies in the teaching-learning processes of probability, and because "there is little attention to the role of intuitions in the understanding of probability in university students" [24], this study set out to explore the perception and obstacles present in students who were taking for the first time courses related to probability and statistics at a Colombian university.

2 METHODOLOGY

The difficulties that students in the early years of higher education face in the study of stochastics are due to the lack of certain intuitive ideas that help them understand and interpret the results they obtain in the construction of central objects of this branch of mathematics [25]. Therefore, it is necessary to identify the perception of the conceptions that are reflected around probability, which has a random, non-deterministic, or futuristic character,

making it difficult to understand for both students and teachers, regardless of the educational level at which stochastics-related topics are taught.

For this research, perception is considered as "Knowledge, idea ", according to the Royal Spanish Academy. This is to identify what students know or understand about the conception of probability according to their cultural or educational experience. It cannot be ignored that probability is "immersed in history, in our culture, and in the society in which we live. Knowledge of probability allows individuals to understand concepts in other areas and answer questions that arise in our society" [19].

The conceptions or meanings of probability are also identified, such as intuitive, frequentist, classical, subjective, and axiomatic [9, 10, 26, 27, 28]. These authors mention that, from a science of education perspective, it is convenient for teachers to instruct their students first with intuitive ideas if the educational level is initial or primary school; then continue with the Laplacian or classical and frequentist conceptions; and finally approach the axiomatic conception in the last school or pre-university years. However, the present research shows the absence of these didactics.

It is worth highlighting that, for the authors mentioned, the conceptions of probability present the following notions: When the intuitive conception is mentioned, it refers to the use of common terms when describing uncertain events (impossible, possible, probable, unlikely, etc.), as a way of measuring the belief that is held of these events [10, 27].

Now, the classical or Laplace conception conceives the quotient between the number of favorable cases for the occurrence of an event on the number of all possible cases, as the probability of said event, only if there is equiprobability in all possible results; this meaning is only conceivable for a finite set of results [9, 10, 27].

When referring to the frequentist conception, or also called frequentist, it is necessary to consider repeating a process many times under the same conditions, and the relative frequency will be the estimate of the probability. Similarly, "probability is defined as the hypothetical number towards which the relative frequency tends to stabilize, assuming the theoretical existence of said limit, of which the observed relative frequency is an approximate value" [25].

On the other hand, the subjective meaning according to [9] "is based on the confidence that a person places on the truth of a certain proposition, so it is not uniquely determined. Probability depends on the observer and what he knows about the event under study"; the author [20] reinforces this meaning by stating that it is "based on the personal experience of each individual, making the probability of an event determined based on what each person believes to be true and more possible, rather than objective fact"; that is, the subjective conception of probability depends on the value judgment that an individual assigns to an event, either by their experience or what they believe may happen regarding said event.

However, when situations arise where the set of results is not

finite, probability is conceived as a special type of measure, linking it to measure theory. This approach establishes axioms to be satisfied and, due to the mathematical rigor involved, it is not recommended for study in primary education. It is a resource of mathematical science that is used to characterize the reality of various stochastic constructs, and it also has a great implication in all areas of human activity. It is worth mentioning that Kolmogorov made a substantial contribution to the formal development of probability [4] and [20].

These meanings of probability should be present in the cerebral structure of students who, like in Colombia, have completed their studies from preschool, primary basic education, secondary, and high school; however, the Colombian mathematics curriculum has been structured in a formal way but with the presence of some weaknesses and erroneous conceptions.

There is consensus that the teaching of probability contributes to citizens understanding the phenomena that arise in society (political purposes), solving situations in everyday life and in the workplace (social purposes), developing knowledge to describe and analyze large amounts of data that allow prediction and decision-making (formative purposes), and recognizing the importance of probability in our culture, its role in history, and in the development of our humanity (cultural purposes) [19].

The study was framed as quantitative research with a descriptive exploratory design. A convenience sample of 313 students who would start a Probability or Statistics course throughout the first and second semesters of 2022 and first of 2023 was selected. A test was administered to them, which included three key questions to diagnose their perception of the conception of probability, the importance of probability in their study programs or in their lives, whether they find it complex to study, and the opportunity to provide comments on its complexity. The students agreed to answer the questions as a basis for starting the development of their course.

The test was reviewed and validated by the researchers and colleagues who have taught these subjects. Once the information was collected and organized, a descriptive-exploratory analysis was carried out to analyze, conclude, and make the corresponding decisions based on the findings of the research.

3 RESULTS

The sample consisted of 313 students, of whom 41.2% were women and 53.4% came from rural institutions. As shown in Figure 1, the students' ages ranged from 18 to 27, with a greater dispersion in the ages of women from rural areas.

According to [26] and [27], it is expected that, at these ages, the presence of the conception of probability would be frequentist, classical, or some idea of the axiomatic conception. However, as shown in Figure 2, 24.9% of the students conceive of probability as a game of chance, 18.8% as the occurrence or non-occurrence of something, and 14.4% perceive it as a method for finding events. In addition, 12.8% of the students present an obstacle by confusing statistical procedures with the concept of probability,

and 14.4% with counting techniques. However, it is recognized that 5.8% of the students present an intuitive conception of probability, with relevance in graduates of urban institutions.
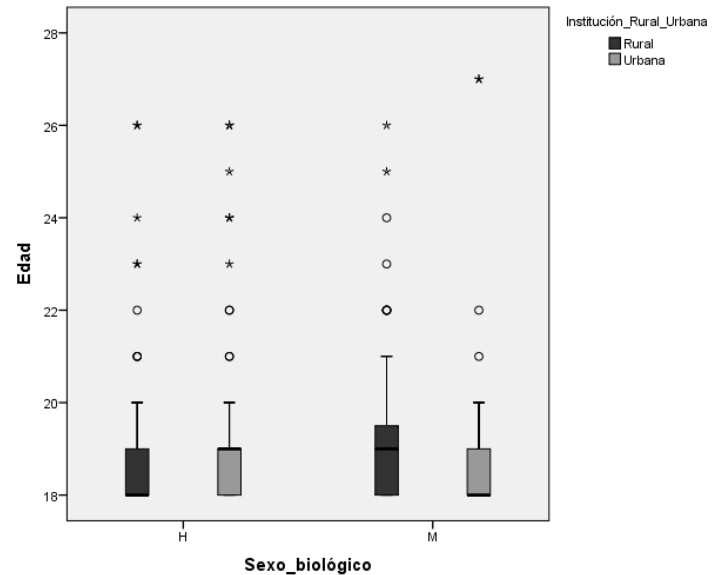

Fig. 1. Age of university students by biological sex and rural or urban institution. Notes: Edad = age; Institución rural Urbana = rural or urban institution; Sexo biológico = biological; H = male; F = Female.

These findings contrast with those of [19] and [24] in that they reflect low levels of understanding of the concept of probability and the entrenchment of obstacles of an epistemological and cognitive nature in relation to the apprehension of the concept. On the other hand, the most common obstacles were found to be related to didactic, epistemological, and ontogenetic factors, and they are attributed to the teacher, the knowledge, the student, and the educational, social, and cultural context. These findings contrast with what is expressed by [19] and show the need for teachers to improve their disciplinary and didactic knowledge through independent learning processes, so that they can guide knowledge around the topic of probability with greater effectiveness in the sense expressed by [9] and [10], among others. These results also showed that there is a relationship between the learning that students had previously presented and the fact that they do not consider probability important in their daily or professional life. In addition, 91.1% believe that it is a very difficult area to study, and that it has been worked very little or almost nothing in their institution. These findings reflect that the topic of probability should continue to be investigated both in its learning and teaching to improve its understanding in the school and university environment. These findings are consistent with [6] and [8] in that diagnoses must involve the implementation of intervention proposals that go beyond probabilistic literacy.
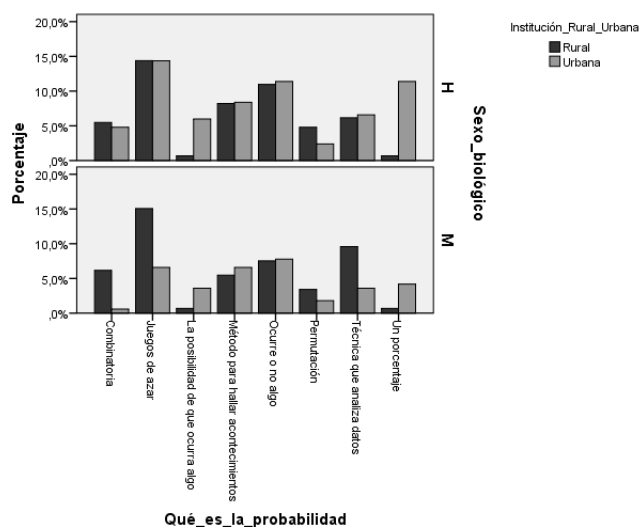
Fig. 2. Perception of the concept of probability of university students by biological sex and rural or urban institution. Notes: Porcentaje=percentage; Sexo biológico= biological sex; H=male; F=female; combinatoria= combinatorics; juegos de azar= games of chance; la posibilidad de que ocurra algo= the possibility that something will happen; Método para hallar acontecimientos= method for finding events; Ocurre o no ocurre algo= something happens or doesn't happen; Permutación= permutation; Técnica que analiza datos= technique that analyzes data; Un porcentaje=a percentage

In this scenario, a methodological proposal consists of developing and applying a didactic sequence that contains specific problem situations about the concept of probability or publish a book with enough allusive situations that include the various conceptions of probability, which allow teachers and students to progressively improve their understanding of the aforementioned concept and move from the conception classical towards the axiomatic by involving both finite and infinite discrete or continuous probability spaces with non-Laplacian measures. These situations must also make it possible to overcome the epistemological, cognitive and didactic obstacles that were identified in the present study, and to improve the inherent curricular aspects.

4 CONCLUSIONS.

In conclusion, the findings of this study show that students do not identify the importance of probability in their academic programs. They also find it very complicated to study. Some students reported that they never reviewed topics related to probability in high school, and even less so the study of the concept itself. Others commented that they rarely touched on probability topics because they were in the last unit of the mathematics program and there was not enough time to cover the topic.

These findings suggest the need for didactic strategies that favor the learning and teaching of probability and their valuable contribution to the curriculum. Recognizing the obstacles that students face in understanding the concept of probability makes it

possible to implement didactic strategies that promote the learning and recognition of probability in teaching practice and in the global context. These findings are a valuable contribution to rethinking and linking the conceptions or notions of probability in a curriculum of its own area or to strengthen the mathematics curriculum.

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