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COST ANALYSIS OF FUZZY DISCRETE (S, S) QUEUEING INVENTORY SYSTEM WITH POSITIVE LEAD TIME AND OPTIMIZATION USING GENETIC ALGORITHM

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Abstract: In this article, a queueing inventory model with discrete time (DQIM) FGEOM/FGEOM/1 with (s, S) replenishment policy incorporating fuzzy numbers as input parameters is considered. The system has a fuzzy pentagonal number arrival rate according to a Bernoulli process and a fuzzy pentagonal number service rate that follows a geometric distribution. Here, S represents the highest level of stock where the process of replenishment is stopped, and s represents the lowest level of stock at which replenishment is started again. Using matrix geometric method, the steady-state solution is obtained followed by derivation of various fuzzy performance measures. Further, the total cost function is defined as a two-variable function of the minimum and maximum stock level. Genetic algorithm is employed to optimize the total cost. Various examples are presented to highlight the dependence of cost on input parameters. The use of PFN in DQIS and genetic algorithm in the optimization of DQIS is introduced in this paper for the first time.

keywords: Discrete Queueing Inventory Miniature (DQIM), Matrix Geometric Method, Positive Lead Time, Pentagonal Fuzzy Number (PFN), Genetic Algorithm.

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Análisis de costos de un sistema de inventario en colas difusas discretas (s, S) con tiempo de entrega positivo y optimización mediante algoritmos genéticos

Resumen: En este artículo se presenta un modelo de inventario en colas con tiempo discreto (DQIM) FGEOM/FGEOM/1 utilizando una política de reabastecimiento (s, S) e incorporando números difusos como parámetros de entrada. El sistema considera una tasa de llegada con un número pentagonal difuso siguiendo un proceso de Bernoulli y una tasa de servicio pentagonal difusa con distribución geométrica. S representa el nivel máximo de inventario en el que se detiene el proceso de reabastecimiento, mientras que s indica el nivel mínimo de inventario a partir del cual se reanuda el reabastecimiento. Empleando el método geométrico matricial, se obtiene la solución en estado estacionario, seguida de la derivación de varias medidas de desempeño difusas. Posteriormente, se define la función de costo total como una función de dos variables que depende del nivel de inventario mínimo y máximo. Se utiliza un algoritmo genético para optimizar el costo total. Se presentan varios ejemplos para destacar la dependencia del costo respecto a los parámetros de entrada. En este trabajo, se introduce por primera vez el uso de números pentagonales difusos (PFN) en el sistema de inventario en colas discretas (DQIS) y la optimización de DQIS mediante algoritmos genéticos.

Palabras clave: Sistema de inventario con colas discretas en miniatura (DQIM), método geométrico matricial, tiempo de entrega positivo, número difuso pentagonal (PFN), algoritmo genético.

1 INTRODUCTION

Inventory control aims to achieve a delicate balance between providing the necessary items while minimizing related costs. Queueing inventory systems offer a range of parameters that can optimize overall system costs and reduce waiting times for customers. In real-life scenarios, such as healthcare systems, telecommunication networks, and traffic systems, time is often measured discretely, despite continuous-time queue, discrete-time queue more favourable.

In this model, discrete type Birth-Death Process (BDP) is used. Here the arrival known as birth follows Bernoulli process, and the departure known as death process follows geometric distribution. Key parameters of the BDP, such as the arrival rate (u) and service rate (v), play a crucial role in determining the optimal inventory level. The arrival process adheres to a Bernoulli distribution with parameter u, resulting in interarrival times that are independent and identically distributed (IID) geometric random variables. Similarly, service times are IID geometric random variables with parameter v and are independent of the arrival process. Two types of Discrete Queueing Inventory Models (DQIM) exist based on the arrival pattern: LAS and EAS. In LAS, arrivals occur just before the slot boundary and before service completions due to occur at the end of that slot (Figure 1A). In EAS, arrivals come early in the slot, just after the slot boundary (Figure 1B).

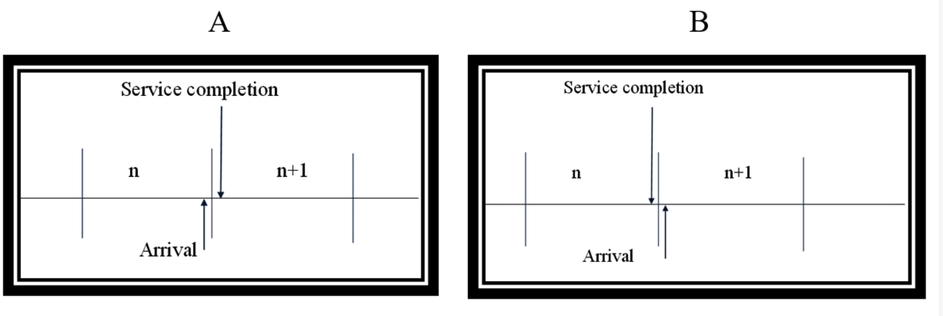

Figure 1. Different type of Arrival Models. **A)** Late Arrival Model and **B)** Early Arrival Model

Several researchers worked for varieties of DQIS. For example, Zhou et al. (2015) analyzed a discrete-time Geo/Geo/1 queueing system with preferred customers and partial buffer sharing. Atencia (2015) explored DQIS with server prone to breakdown. Additionally, Atencia (2017) discussed retrial DQIS and with priority services. A comparison of two DQIS with different policies queue dependent and inventory level dependent arrivals was presented by Balagopal et al. (2020). Lan and Tang (2017) analyzed the performance measures of DQIS with working vacations. Jeganathan et al. (2023) discussed very well server vacation and customer retrial facilities in a multi-server queueing inventory system. Abdali et al. (2024) focused on customer prioritization using queueing-inventory approach for inventory management in multi-channel service retailing with cost optimization using machine learning algorithms. Furthermore, Krishnamoorthy et al. (2024) described QIS with simulation modelling of classical and retrial queues.

In QIS we can consider two types of replenishment policy (s, S) policy and (s, Q) policy. In both cases the lowermost level of stock is denoted by s, as the inventory level drops to s, new replenishment is done. In (s, S) policy if the inventory level become x<s then replenishment is of (S-x) and in (s, Q) policy, the replenishment is of order Q where Q = S – s. Balagopal et al. (2021) compared two DQIS with (s, S) and (s, Q) policies and presented a cost analysis. Akar and Dogan (2021) described the use of DQIS in parameters in a practical IoT system. Anilkumar and Jose (2022) employed a matrix geometric method to analyze a discrete-time (s, S) queueing inventory model with back-order in inventory. Kempa and Paprocka (2024) used a discrete-time queueing model to simulate a sustainable production system.

In many studies, researchers use fuzzy parameters in DQIS to incorporate fuzziness of real-world scenarios (Lan & Tang, 2017). Fuzzy numbers are used in M/M/s multi-server systems (Narayanamoorthy & Ramya, 2017). The application of PFN to describe QIS with multi-server model have been explored in various studies (Visalakshi & Suvitha, 2018; Jain & Jain, 2021).

The genetic algorithm is based on the abstraction of real biological evolution system. To decide the input parameters to optimize overall cost of system use of genetic algorithm is best option among all classical optimization method. It is used to find better solution among all possible solution. Some researchers use GA in Markovian queueing system with Queue based arrival rates (Ke et al., 2010). Use of genetic algorithm in M/M/s to optimize costs has been thoroughly explained in many studies (Xu et al., 2014; Jain & Jain, 2022).

This paper investigates the optimization of a discrete time queueing inventory miniature FGEOM/FGEOM/1 with a positive lead time, where arrival time and service time are modelled using a Bernoulli process. The input parameters are treated as PFN fuzzy numbers and analysed in a fuzzy environment. The steady state equations are formulated using the matrix geometric method and solved using arithmetic of PFN to obtain the probability vector of PFNs. Fuzzy Execution parameters are derived, and a cost formula is developed to account for various factors affecting the overall cost of the system. Genetic algorithm is applied to find the optimal values of parameters for optimum cost. The results reveal the sensitivity of performance measures to various parameters in a fuzzy sense. Numerical examples are presented, including sensitivity analysis, to illustrate the approach.

The systematic flow of the matter of the paper is as follows. In Section 2, we introduce fuzzy sets and PFN (pentagonal fuzzy number), as well as arithmetic operations involving PFN. In Section 3, we explain our proposed model in detail. Section 4 presents the results of our steady-state analysis, where we use the matrix geometric method to derive stationary probabilities. In section 5 we discuss various execution features of our proposed model. Next, in Section 6, we present a formula for the overall cost of the system, which is a function of several performance measures. We use graphs to demonstrate some numerical results, where input parameters and output measures are represented as PFNs. In Section 7, we discuss the cost optimization of our proposed model using a GA. Finally, we conclude our paper and suggest possible avenues for further improvement in Section 8.

2 BASIC FUZZY SETS

Zadeh introduced the concept of Fuzzy sets in 1965 to address the uncertainties that arise in our daily lives. To handle the arithmetic complexities of fuzzy variables, fuzzy numbers were introduced.

Fuzzy set: A fuzzy set can be represented as ordered pairs defined on the real line, as $\hat{A} = \{(a, \mu(a)), a \in R\}$, where the degree of membership is determined by membership function $\mu(a)$.

α-cut: An α-cut denotes a set of members from universe U with there $\mu(a) > \alpha$.

A fuzzy number: A fuzzy number is a specific kind of fuzzy set defined on the real line that meets the criteria of normality and convexity. When the membership function of a fuzzy set is piecewise continuous, it is referred to as a fuzzy number.

2.1 PFN (pentagonal fuzzy number):

A fuzzy number $\tilde{F} = (f_1, f_2, f_3, f_4, f_5)$ with its membership function given below (equation 1) is called PFN

$$\mu_A(f, w_1, w_2) = \begin{cases} 0 & \text{for } f < f_1 \text{ or } f > f_5; \\ w_1\left(\dfrac{f - f_1}{f_2 - f_1}\right) & \text{for } f_1 \le f \le f_2; \\ 1 - (1 - w_1)\left(\dfrac{f - f_3}{f_2 - f_3}\right) & \text{for } f_2 \le f \le f_3; \\ 1 & \text{for } f = f_3; \\ 1 - (1 - w_2)\left(\dfrac{f - f_3}{f_4 - f_3}\right) & \text{for } f_3 \le f \le f_4; \\ w_2\left(\dfrac{f - f_5}{f_4 - f_5}\right) & \text{for } f_4 \le f \le f_5. \end{cases} \quad (1)$$

Each PFN is linked to two weight functions, denoted as w_1 and w_2, which range between 0 and 1. When w_1 and w_2 both equal 0, the PFN is a triangular fuzzy number, while when w_1 and w_2 equal 1, the PFN becomes a trapezoidal number.

2.2 Arithmetic of two PFNs

Let $Y=(y_1,y_2,y_3,y_4,y_5)$ and $Z=(z_1,z_2,z_3,z_4,z_5)$ be two PFNs, Various Arithematic operations are shown below (equations 2 to 6)

$$Y+Z=(y_1+z_1,y_2+z_2,y_3+z_3,y_4+z_4,y_5+z_5) \tag{2}$$

$$Y-Z=(y_1-z_5,y_2-z_4,y_3-z_3,y_4-z_2,y_5-z_1) \tag{3}$$

$$Y.Z=(k_1.z_1,k_2.z_2,k_3.z_3,k_4.z_4,k_5.z_5) \tag{4}$$

$$\frac{Z}{Y}=\left(\frac{z_1}{y_5},\frac{z_2}{y_4},\frac{z_3}{y_3},\frac{z_4}{y_2},\frac{z_5}{y_1}\right) \tag{5}$$

$$\frac{1}{Y}=\left(\frac{1}{y_5},\frac{1}{y_4},\frac{1}{y_3},\frac{1}{y_2},\frac{1}{y_1}\right) \tag{6}$$

2.3 Model description

This article focuses on DQIS with (s, S) policy and with positive lead times. We can consider it as a specific instance of BDP. We examine the (s, S) replenishment policy in this model where s minimum and maximum inventory levels are denoted by s and S respectively. In our EAS system, arrivals occur in the interval (n, n+1) and departures occur in the interval (n−1, n). To prevent continuous reordering, we set the S > 2s. Demand according to Bernoulli process, while service rates and lead times according to geometric distribution. This paper examines the FGeom/FGeom/1 (s, S) discrete inventory system with the following input parameters. The demand follows Bernoulli process with parameter u as fuzzy pentagonal number denoted by u. Also, service time v and the lead time w follows geometric distributions as fuzzy pentagonal number.

Let us consider the joint inventory process $\Psi=\{(N_m,I_m),m\in N\}$ where N_m,I_m represents the count of customer and inventory level of the system respectively, with the state space C = {0, 1, 2, }×{0,1,…,s,s+1,…S}. Let the transition matrix for this process be M as shown below (equation 7):

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$$M = \begin{bmatrix} M_{00} & M_{01} & & & & & & \\ M_{10} & M_{11} & M_{12} & & & & & \\ & M_{10} & M_{11} & M_{12} & & & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & M_{10} & M_{11} & M_{12} & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & \ddots & \ddots & \ddots \\ & & & & & & & \end{bmatrix} \tag{7}$$

The transitions at level 0 are denoted by M_{00} of dimension S+1. M_{01} denotes the shifts from level 0 to level 1. Submatrices of the matrix M are generated as (equations 8 to 13):

$$[M_{00}]_{ab} = \begin{cases} \widetilde{w}, & a = 0,\ b = a; \\ \tilde{u}\widetilde{w}, & a = 1,2,\dots,s,\ \ b = a; \\ \tilde{u}\widetilde{w}, & a = s+1, s+2, \dots S,\ \ b = a; \\ \tilde{u}w, & a = 0,1,\dots,s,\ b = a; \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

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$$[M_{10}]_{ab} = \begin{cases} \tilde{u}vw, & a = 1,\ldots,s, b = S-1; \\ v\tilde{w}, & a = 1, b = a-1; \\ \tilde{u}\tilde{w}, & a = 2,3,\ldots,s, b = a-1,\ ; \\ \tilde{u}v, & a = s+1, s+2, \ldots, S, b = a-1; \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

$$[M_{01}]_{ab} = \begin{cases} uw, & a = 0,1,2,3,\ldots,s, b = S; \\ u\tilde{w}, & a = 1,2,3,\ldots,s, b = a; \\ u, & a = s+1, s+2, \ldots S,\ b = a; \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

$$[M_{12}]_{ab} = \begin{cases} u\tilde{v}w, & a = 0,1,\ldots,s\ \ b = S; \\ u\tilde{v}\tilde{w}, & a = 1,2,\ldots,s,\ b = a; \\ u, & a = s+1, s+2, \ldots S-1,\ b = a; \\ uw, & a = 0,\ b = S; \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

$$[M_{11}]_{ij} = \begin{cases} uvw, & a = 1,\ldots,s,\ b = S-1; \\ \tilde{w}, & a = 0,\ b = a; \\ \tilde{u}\tilde{v}\tilde{w}, & a = 1,2,\ldots,s, b = a; \\ \tilde{u}\tilde{v}, & a = s+1, s+2, \ldots S-1,\ b = a; \\ uv\tilde{w} & a = 2,3,\ldots,s,\ b = a-1; \\ \tilde{u}w, & a = 0,\ b = S; \\ \tilde{u}\tilde{v}w & a = 1,2,\ldots,s,\ b = S; \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

Here $\tilde{u}, \tilde{v}$ and $\tilde{w}$ denotes complimentary probabilities.

3 STEADY STATE ANALYSIS

Let us consider$T = M_{10} + M_{11} + M_{12}$. The joint inventory process Ψ will be stable if $\tilde{y}M_{12}e < \tilde{y}M_{10}e$ here

$\tilde{y} = (\tilde{y}_0, \tilde{y}_1, \tilde{y}_2, \ldots \tilde{y}_s, \ldots, \tilde{y}_S)$ denotes SPV (stationary probability vector).

The vector y satisfies the following conditions for steady state (equation 14):

$$\tilde{y}T = \tilde{y}, \tilde{y}e = 1 \tag{14}$$

Equation 8 further reduces into (equations 15 to 16):

$$\tilde{\pi}_k = \begin{cases} \dfrac{(1-\widetilde{w})(1-\tilde{v}\widetilde{w})^s}{u(u\widetilde{w})^s}\tilde{\pi}_0, & k > s; \\ \dfrac{(1-\widetilde{w})(1-\tilde{v}\widetilde{w})^{k-1}}{(u\widetilde{w})^k}\tilde{\pi}_0, & k = 1,2,\ldots,s; \end{cases} \tag{15}$$

$$\tilde{\pi}_0 = \frac{\tilde{u}v(v\widetilde{w})^s}{(1-\tilde{v}\widetilde{w})^s[\tilde{u}v + (S-s-1)w + w\tilde{v}] + wv(v\widetilde{w})^s} \tag{16}$$

On solving $yM = y \; and \; ye = 1$ we get (equations 17 to 19)
where $y = (y_0, y_1, y_2, \ldots)$ is SPV.

$$y_0 M_{00} + y_1 M_{10} = y_0 \tag{17}$$

$$y_0 M_{01} + y_1 M_{11} + y_2 M_{12} = y_1 \tag{18}$$

$$y_{n-1} M_{12} + y_n M_{11} + y_{n+1} M_{10} = y_n, n \geq 2 \tag{19}$$

Using $y_n = y_1 \bar{R}^{n-1}, n \geq 2$ where $\bar{R}$ is obtained using the equations 20 to 21:

$$M_{12} + \bar{R} M_{11} + \bar{R}^2 M_{10} = \bar{R} \tag{20}$$

$$y_0 e + y_1 (1-\bar{R})^{-1} e = 1 \tag{21}$$

Recursive mechanism is used to find $\bar{R}$ as shown below (equation 22):

$$\bar{R}(0) = 0. \tag{22}$$

$$\text{while} |\bar{R}(n) - \bar{R}(n+1)|_{ij} < \varepsilon$$

$$\bar{R}(n+1) = (M_{12} + \bar{R}(n)^2 M_{10})(1 - M^{11})^{-1} \tag{23}$$

Let $y = \rho\tilde{y}$ and $y_n = \rho\left(\frac{u}{\tilde{u}v}\right)\tilde{y}$ where $\rho = 1 - \frac{u}{\tilde{u}v}$ then the SPV $y = (y_0, y_1, y_2, \ldots)$ can be obtained $y_n = (y_{n0}, y_{n1}, y_{n2}, \ldots, y_{nS})$.

4 PERFORMANCE MEASURES

Functioning measure (equations 24 to 29) can be obtained using the SPV y as:

Number of demands:

$$\mathbb{N}_{cust} = \sum_{n=0}^{\infty} n y_n e \tag{24}$$

Loss rate:

$$\mathbb{N}_{loss} = u \sum_{n=0}^{\infty} n y_{n0} \tag{25}$$

Staying consumers at zero stock level

$$\mathbb{N}_{CW} = \sum_{n=0}^{\infty} n y_{n0} \tag{26}$$

Replenishment rate:

$$\mathbb{N}_{repl} = w \sum_{n=0}^{\infty} \sum_{m=0}^{s} y_{nm} \tag{27}$$

Reorder rate:

$$\mathbb{N}_{record} = v \sum_{n=0}^{\infty} y_{n,s+1} \tag{28}$$

Level of stock:

$$\mathbb{N}_{inv} = \sum_{n=0}^{\infty} \sum_{m=1}^{S} m y_{nm} \tag{29}$$

5 COST ANALYSIS

Let us consider the overall cost associated with given model (equation 30):

$$T_{cos\,t} = \left[F + \sum_{i=0}^{s} w(S-i)\, P \right] \mathbb{N}_{record} + H\mathbb{N}_{inv} + C\mathbb{N}_{cw} + L\mathbb{N}_{loss} \tag{30}$$

Here F, P, H, C and L are fixed cost are as follows:

F: fixed, P: purchasing, H: holding item per unit time, C: holding customers per unit time, L: loss per customer.

5.1 Mathematical illustration:

Sample 1: Let us consider arrival rate u = (0.11, 0.12, 0.13, 0.14, 0.15), service rate v= (0.61, 0.62, 0.63, 0.64, 0.65), and lead time w= (0.21, 0.22, 0.23, 0.24, 0.25), respectively as PFN. Let other parameters S=11, s=5 and F=55, P=55, H=1.1, C=2.1, L=3.1 as crisp numbers. Now on solving the equation 8-18 using MATLAB programming with fuzzy arithmetic various performance measures are calculated. The results are graphically presented below (Figure 2-3):

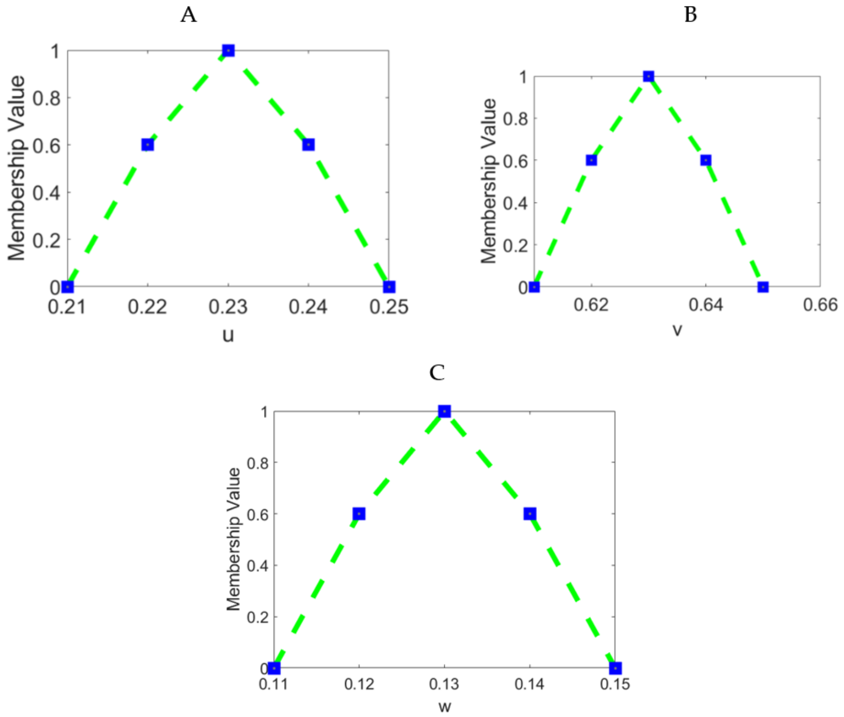

Figure 2. Various Inputs. **A)** Arrival rate as PFN, **B)** Service rate as PFN and **C)** Lead time as PFN

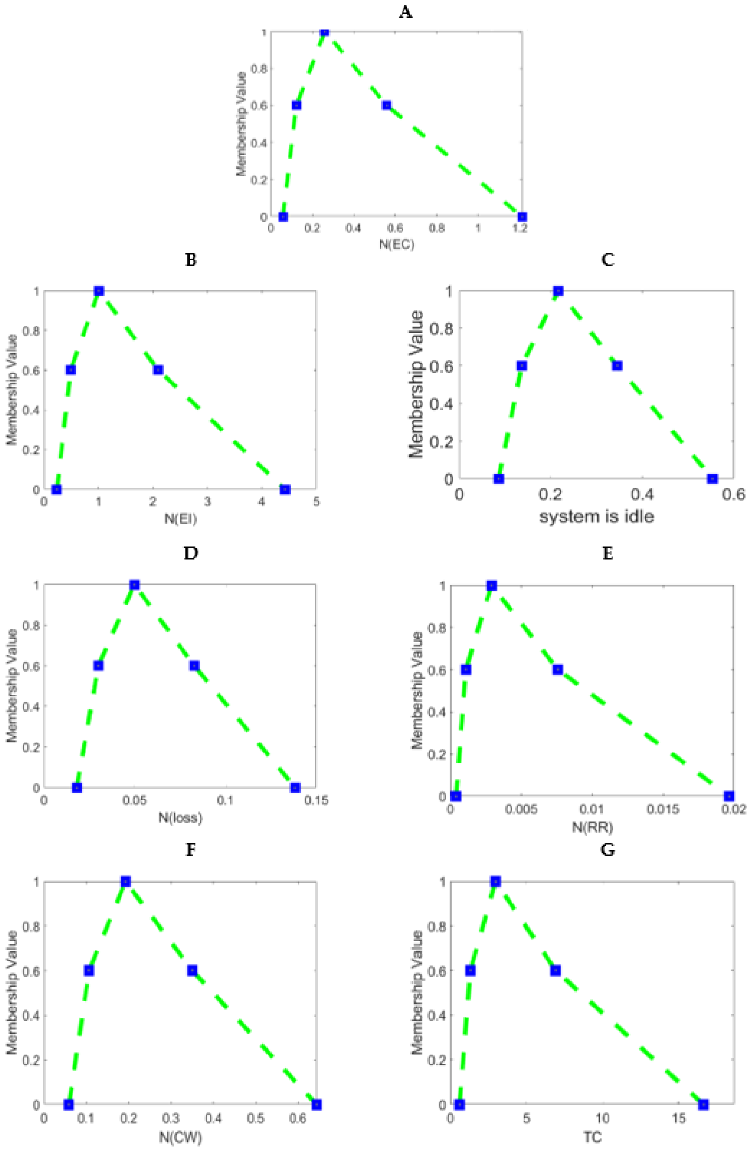

Figure 3. Effect of input parameters on various performance measures. **A)** Waiting customers at zero inventory level, **B)** Inventory level, **C)** Probability that system is idle, **D)** Average loss of customer, E) Reorder Rate, **F)** Average waiting customer at zero inventory level and **G)** Total cost.

Sample 2:

Let us consider arrival rate u = (0.11, 0.12, 0.13, 0.14, 0.15), service rate v= (0.71, 0.72, 0.73, 0.74, 0.75), and lead time w= (0.21, 0.22, 0.23, 0.24, 0.25), respectively as PFN. Let other parameters S=11, s=5 F=55, P=55, H=1.1, C=2.1, L=3.1as crisp numbers. Now on solving the equation 8-18 using MATLAB programming with fuzzy arithmetic various performance measures are calculated. The results are graphically presented below (Figure 4-5):

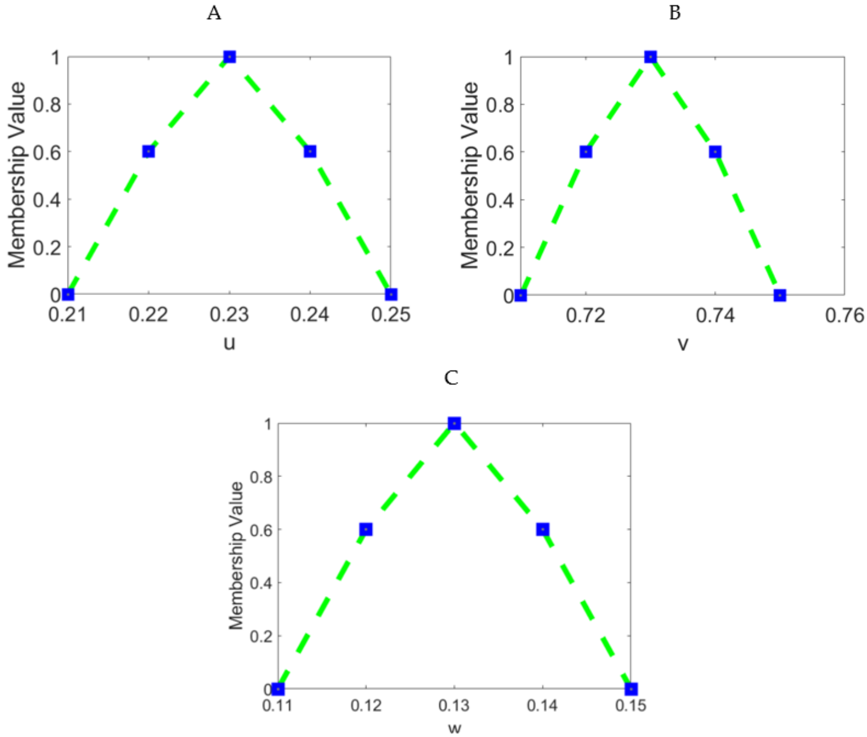

Figure 4. Various Inputs. **A**) Arrival rate as PFN, **B)** Service rate as PFN and **C)** Lead time as PFN

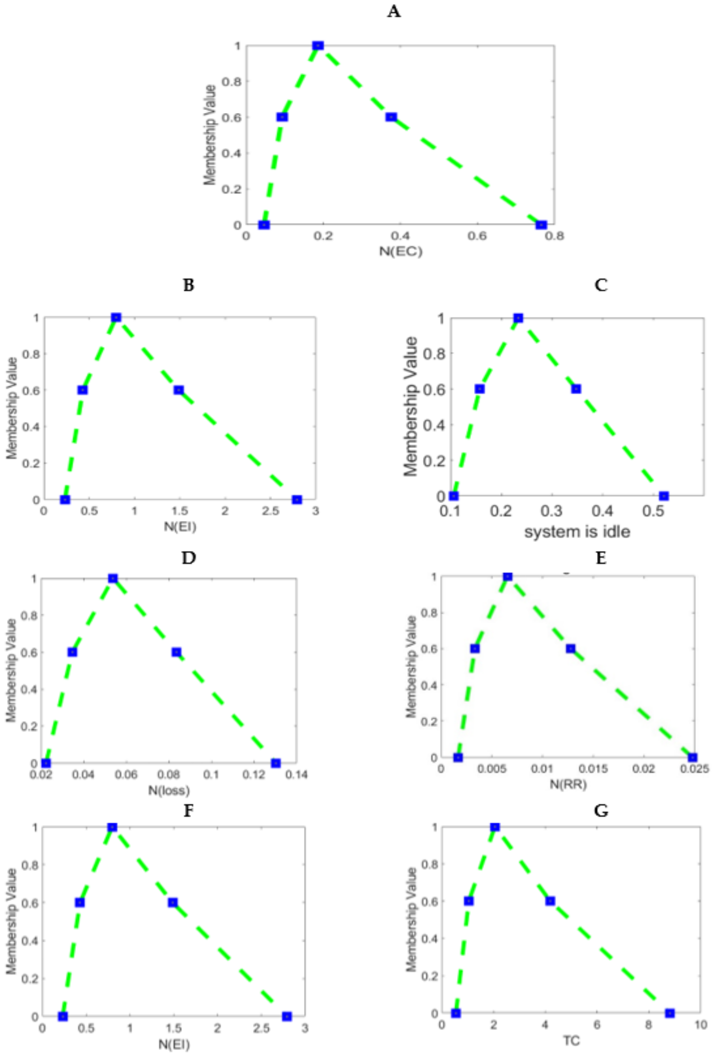

Figure 5. Effect of input parameters on various performance measures. **A)** Waiting customers at zero inventory level, **(B)** Inventory level, **C**) Probability that system is idle, **D**) Average loss of customer, **E)** Reorder Rate, **F)** Average waiting customer at zero inventory level and **G)** Total cost.

Sample 3:

Let us consider arrival rate u = (0.11, 0.12,0.13,0.14,0.15), service rate v= (0.61, 0.62, 0.63, 0.64, 0.65), and lead time w= (0.26, 0.27, 0.28, 0.29, 0.30), respectively as PFN. Let other parameters S=11, s=5 and F=55, P=55, H=1.1, C=2.1, L=3.1 as crisp numbers. Now on solving the equation 8-18 using MATLAB programming with fuzzy arithmetic various performance measures are calculated. The results are graphically presented below (Figure 6-7):

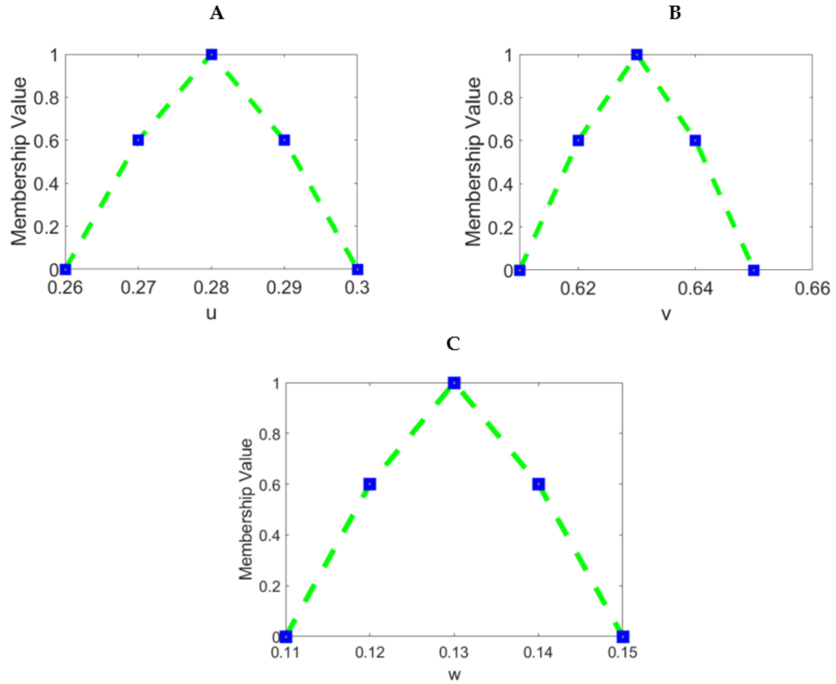

Figure 6. Various Inputs. **A)** Arrival rate as PFN, **B)** Service rate as PFN and **C)** Lead time as PFN

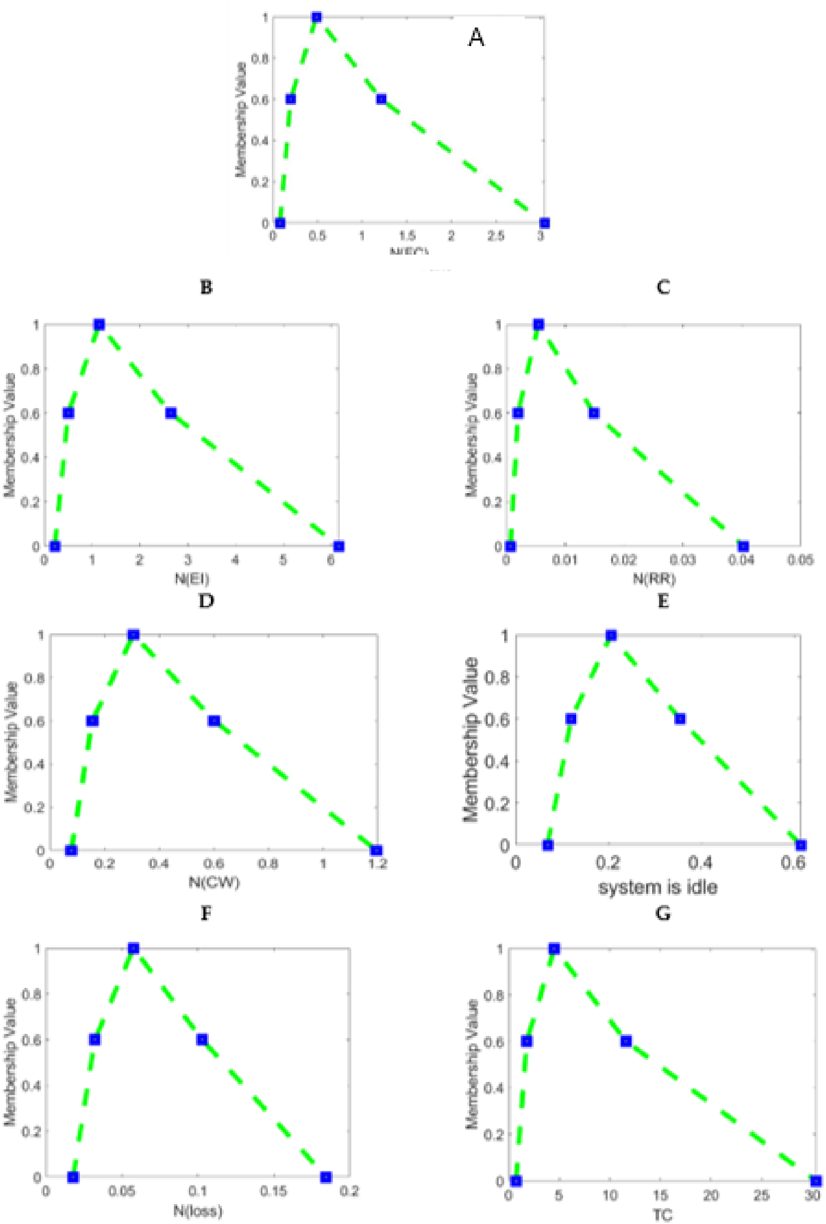

Figure 7. Effect of input parameters on various performance measures. **A**) Waiting customers at zero inventory level, **B)** Inventory level, **C)** Probability that system is idle, **D)** Average loss of customer, **E)** Reorder Rate, **F)** Average waiting customer at zero inventory level and **G)** Total cost.

The results can be summarized as follows (Table 1):

Table 1. Sensitivity analysis

	Sample 1	**Sample 2**	**Sample 3**
u	(0.21, 0.22, 0.23, 0.24, 0.25)	(0.21, 0.22, 0.23, 0.24, 0.25)	(0.26, 0.27, 0.28, 0.29, 0.30)
v	(0.61, 0.62, 0.63, 0.64, 0.65)	(0.71, 0.72, 0.73, 0.74, 0.75)	(0.61, 0.62, 0.63, 0.64, 0.65)
w	(0.11, 0.12, 0.13, 0.14, 0.15)	(0.11, 0.12, 0.13, 0.14, 0.15)	(0.11,0.12,0.13,0.14,0.15)
N[EC)	[0.057, 0.127, 0.289, 0.637, 1.454)	(0.597, 1.308, 2.962, 6.921, 16.636)	(0.080, 0.198, 0.488, 1.211, 3.043)
N(IDLE)	(0.086, 0.136, 0.216, 0.344, 0.553)	(0.106, 0.157, 0.233, 0.347, 0.520)	(0.069, 0.119, 0.205, 0.354, 0.614)
N(EI)	(0.239, 0.489, 1.008, 2.096, 4.420)	(0.232, 0.430, 0.797, 1.486, 2.792)	(0.218, 0.502, 1.148, 2.643, 6.155)
N(LOSS)	(0.018, 0.030, 0.050, 0.083, 0.138)	(0.022, 0.034, 0.053, 0.083, 0.130)	(0.0179, 0.032, 0.0576, 0.102, 0.184)
N(RR)	(0.0003.0010, 0.0028 ,0.0075, 0.0196)	(0.0002, 0.0006, 0.0014, 0.0036, 0.0085)	(0.0006, 0.001, 0.005, 0.0148, 0.040)
N(CW)	(0.059, 0.106, 0.192, 0.350, 0.643)	(0.058, 0.096, 0.161, 0.269, 0.455)	(0.078, 0.155, 0.306, 0.603, 1.196)
COST	(0.597, 1.308, 2.962, 6.921, 16.636)	(0.542, 1.039, 2.053, 4.187, 8.805)	(0.7283, 1.799, 4.525, 11.607, 30.355)

5.1 Cost optimization using GA

The genetic algorithm is a computerized method inspired by biological evolution that seeks to find the optimum solution from a given population of solution. It is particularly well-suited for optimizing multivariable functions, outperforming classical optimization techniques. The following algorithm outlines the algorithm:

Create an initial set of candidate solutions, called chromosomes.

Using cost function evaluates the fitness of each chromosome.

Select the best-fit chromosomes.

Reproduce a new population through mutation and crossover operators.

New offspring are incorporated into the new population after evaluating their fitness.

Set the stopping criteria, repeat steps 2-5 if condition fails.

The above algorithm is executed in MATLAB, where input parameters u, v, and w are assigned random values using the random function as crisp numbers. The cost function, defined using a MATLAB program, is a function of these parameters and is used to evaluate the fitness. Using fixed input parameters S=11, *s*=5, F=100, P=50, H=.2, C=.3, L=.5 and population of chromosomes using different values of input parameters *u*, *v*, *w* we apply genetic algorithm to find optimum value of overall cost as shown below (Table 2).

Table 2. Optimization using genetic Algorithm

Total cost	***u***	***v***	***w***
43.67532	0.369	0.843	0.081
59.88227	0.132	0.958	0.084
68.95311	0.311	0.771	0.257
71.85285	0.106	0.896	0.203
94.10829	0.21	0.935	0.278
96.85476	0.178	0.802	0.331
98.63243	0.203	0.726	0.377
101.9572	0.353	0.805	0.191
108.8475	0.127	0.741	0.4
125.9954	0.188	0.778	0.229

6 CONCLUSIONS

This paper proposes a novel fuzzy discrete queueing inventory model, specifically FGEOM/FGEOM/1, with a replenishment policy of (s, S). The model employs fuzzy pentagonal numbers to enhance the accuracy of the discrete queueing system. Moreover, the study introduces genetic algorithms to DQIS for the first time. To use PFN as input parameters, steady-state probabilities have been calculated by solving the QBD process equations. Then, PFN arithmetic is used to derive performance measures for fuzzy input parameters. In addition, stationary probabilities h calculated using the matrix geometric method to find crisp values, from which further performance measures are derived. A cost formula is developed as a function of crisp input parameters, such as arrival rate, service rate, and lead time. Using GA, the study searches for the best values of these parameters to minimize the cost. This research provides significant benefits for engineers and managers in managing economic systems. Future studies could extend this research by applying cost optimization with GA for multi-server DQIS and specific cases using DQIS.

AUTHORS' CONTRIBUTIONS

Mridula Jain: Conceptualization; Methodology; Validation; Formal analysis; Data curation; Resource acquisition; Investigation; Methodology; Software; Display; Writing (original draft). Indeewar Kumar: Project management; Resources; Supervision; Validation; Writing (draft review and revision/correction). All authors have read and accepted the published version of the manuscript.

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The authors declare that they have no conflicts of interest.

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