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APPLICATION OF NEW FUZZY MEASURE IN MULTI-ATTRIBUTE DECISION-MAKING

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Abstract: The implementation of multi-attribute decision-making (MADM) and the different approaches that facilitate it are addressed in this article. We focus on a recently developed fuzzy divergence measure, which is critical in improving decision accuracy when faced with several conflicting criteria. To highlight its real-world relevance, we present a detailed case study focusing on picking the best market for investment. In this case study, the previously studied Fuzzy divergence measure that is used to evaluate and prioritize various market possibilities based on important characteristics such as risk, return, and market potential. In this example, we demonstrate how this unique measure improves decision-making processes by providing a more precise and comprehensive method to selecting the greatest investment possibilities in uncertain and complicated contexts. The findings highlight the measure's usefulness in guiding investment decisions and enhancing the overall efficacy of MADM applications.

Palabras clave: New fuzzy divergence measure, Properties, Multi Attribute Decision making, Numerical presentation.

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Aplicación de la nueva medida difusa en la toma de decisiones multiatributos

Resumen: En este artículo se aborda la implementación de la toma de decisiones multiatributo (MADM) y los diferentes enfoques que la facilitan. Nos centramos en una medida de divergencia difusa desarrollada recientemente, que es fundamental para mejorar la precisión de las decisiones cuando se enfrentan varios criterios contradictorios. Para resaltar su relevancia en el mundo real, presentamos un estudio de caso detallado que se centra en elegir el mejor mercado para invertir. En este estudio de caso, la medida de divergencia difusa previamente estudiada se utiliza para evaluar y priorizar varias posibilidades de mercado en función de características importantes como el riesgo, el rendimiento y el potencial de mercado. En este ejemplo, demostramos cómo esta medida única mejora los procesos de toma de decisiones al proporcionar un método más preciso y completo para seleccionar las mayores posibilidades de inversión en contextos inciertos y complicados. Los hallazgos resaltan la utilidad de la medida para guiar las decisiones de inversión y mejorar la eficacia general de las aplicaciones MADM.

Palabras clave: Nueva medida de divergencia difusa, propiedades, toma de decisiones multiatributo, presentación numérica.

1 INTRODUCTION

In addition to outlining numerous mathematical concepts pertinent to both types of distributions, Shannon (1948) constructed the measure of ambiguity for discrete and continuous probability distributions. Along with discovering fuzzy set theory in 1965, Zadeh (1962,1968, 2002) created fuzzy entropy in 1962, that is a fuzzy information measure inspired by Shannon's entropy. To construct fuzzy entropy, De Luca and Termini (1972) proposed an array of circumstances. For fuzzy sets, Bhandari and Pal (1993) presented a number of variables that include a divergence measure.

The information theoretic metric of discrimination between two sets that they also supplied is the same as the uncertain entropy measure that De Luca and Termini created (1972). The fuzzy set's random entropy with different properties has been extended from Rényi's (1961) probabilistic entropy of order α. Applications such as image processing and clustering have made use of these metrics. Subsequently, investigators like Montes et al. (2002), Pal and Bezdek (1994), Parkash and Gandhi (2010), Criado and Gachechiladze (1997), Pal and Pal (1989), Yager (2001), Bajaj and Hooda (2010), Kahraman (2008), Verma and Dev (2015), Umar and Saraswat (2021), and Verma and Sharma (2011), among others, have also put out a number of entropies, divergence metrics, and entropy generalizations with related characteristics.

The role of divergence measures becomes especially critical in multi-attribute decision making (MADM) (Celik et al., 2019), where decisions must be made based on multiple, often conflicting, criteria. Divergence measures, which quantify the dissimilarity between sets of information or criteria, play a vital role in evaluating and distinguishing between options in decision-making processes. In MADM, particularly in fuzzy contexts where information can be imprecise or uncertain, fuzzy divergence measures help decision-makers assess the degree of similarity or difference between alternatives. This is crucial for comparing options and assigning relative importance to different criteria. Fuzzy MADM methods incorporate fuzzy divergence measures—like those developed by De Luca and Termini (1972)—to handle ambiguous or incomplete information, allowing decision-makers to express preferences in linguistic terms, such as "high," "medium," or "low."

By using these divergence measures, decision-makers can more accurately evaluate the uncertainty associated with each option and make more informed decisions. The application of fuzzy divergence in MADM highlights the need for reliable measures of dissimilarity to ensure effective prioritization and selection of alternatives. Therefore, the development and refinement of these measures are integral to improving decision-making frameworks. This work explores the application of divergence measures in MADM, underscoring their significance in evaluating alternatives and demonstrating their validity in real-world decision-making scenarios.

Based on the previous discourse, we are able to approximate the significance of measures of divergence and dissimilarity. We employ different divergence measures based on the condition. It is therefore essential to hunt for new measures in the previous piece that are effective and reliable performers. This work focuses on the use of divergence measures in MADM, with previously mentioned qualities and demonstration of their validity.

Section 2 of this paper provides various definitions and concepts that are necessary. Utilizing the divergence measure in MADM is covered in Section 3, and Section 4 presents an example. Section 5 offers as the paper's ultimate conclusion.

2 PRELIMINARY

In this section, we present offers a variety of essential definitions and a plan for dealing with MADM problems.

2.1 Fuzzy Set:

When Z is a discourse universe and z is any specific member within Z, a fuzzy set defined on Z can be characterized as an aggregation of ordered. (Ohlan ,2015)

$$₳ = \{\langle z, \upsilon_{₳}(z) | z \in Z\rangle\} \qquad (1)$$

In this scenario, any pair $(z, \upsilon_{₳}(z))$ will be referred to as a singleton, and $\upsilon_{₳}$ is the fuzzy set ₳'s membership function, or degree of belongingness, that is determined by the subsequent equation:

$$\upsilon_{₳}(z): Z \rightarrow [0,1]$$

One possible approach for managing uncertainty and imprecision is the fuzzy set.

A fuzzy set ₳ is convex if

$$\upsilon_{₳}(\lambda z_1 + (1-\lambda)z_2) \geq \min\{\upsilon_{₳}(z_1), \upsilon_{₳}(z_2)\}\ , z_1, z_2 \in \ Z, \lambda \in [0,1]. \qquad (2)$$

Conversely, if each collection at the α-level is convex, then a fuzzy set is convex.

2.2 Operations on Fuzzy Sets

Zadeh (2002) defined the following operations for fuzzy sets as generalization of crisp sets and of crisp statements (the reader should realize that the set theoretic operations intersection, union and complement correspond to the logical operators and, inclusive or and negation)

Intersection (Logical and): The following defines the membership function of the point where two fuzzy sets, ₳ and ₿, intersect:

$$\upsilon_{₳\cap ₿}(z) = \min\{\upsilon_{₳}(z), \upsilon_{₿}(z)\} \forall z \epsilon Z\ . \qquad (3)$$

Union (Exclusive or): The union defines its membership function as:

$$\upsilon_{\text{A}\cup\text{B}}(z) = \max\{\upsilon_{\text{A}}(z), \upsilon_{\text{B}}(z)\}\ \forall z \epsilon Z\ . \qquad (4)$$

Complement (Negation): The complement's membership function is described as follows:

$$\upsilon_{\overline{\text{A}}}(z) = 1 - \upsilon_{\text{A}}(z) \forall z \epsilon Z \qquad (5)$$

2.3 New Proposed Measure and It's Properties

A divergence measure is introduced by Jain and Chhabra (2014) and is shown as follows:

$$P^*(M,N) = \frac{1}{2}\sum_{i=1}^{p} \frac{(m_i - ni)^4}{(m_i + n_i)^3}. \qquad (6)$$

For a convex and normalized function $\varphi: (0,\infty) \to R$ (a set of real no) defined as:

$\varphi(t) = \frac{(t-1)^4}{t^3}$, t$\epsilon(0,\infty)$ and $\varphi(1) = 0$.

Here, we propose a measure for comparing the dissimilarities between the two fuzzy sets A and B

found in the domain of discourse Z.

It is given by

$$\text{Ҕ}(\text{A},\text{B}) = \frac{1}{2}\sum_{i=1}^{p} (\upsilon_{\text{A}}(z_i) - \upsilon_{\text{B}}(z_i))^4 \left[\frac{1}{\left(\upsilon_{\text{A}}(z_i) + \upsilon_{\text{B}}(z_i)\right)^3} + \frac{1}{\left(2 - \upsilon_{\text{A}}(z_i) - \upsilon_{\text{B}}(z_i)\right)^3}\right]. \qquad (7)$$

Following are its characteristics, which have already been established in our prior article (Chhabra & Chhabra, 2023):

- **a)** $\text{Ҕ}(\text{A}\cup\text{B}, \text{A}\cap\text{B}) = \text{Ҕ}(\text{A},\text{B})$
- **b)** $\text{Ҕ}(\text{A}\cup\text{B}, \text{A}) + \text{Ҕ}(\text{A}\cap\text{B}, \text{A}) = \text{Ҕ}(\text{A},\text{B})$
- c) $\text{Ҕ}(\text{A}\cup\text{B}, \boldsymbol{C}) \leq \text{Ҕ}(\text{A}, \boldsymbol{C}) + \text{Ҕ}(\text{B}, \boldsymbol{C})$
- **d)** $\text{Ҕ}(\text{A}\cup\text{B}, \boldsymbol{C}) + \text{Ҕ}(\text{A}\cap\text{B}, \text{A}) = \text{Ҕ}(\text{A}, \boldsymbol{C}) + \text{Ҕ}(\text{B}, \boldsymbol{C})$
- e) $\text{Ҕ}(\text{A}, \text{A}\cap\text{B}) = \text{Ҕ}(\text{B}, \text{A}\cup\text{B})$
- **f)** $\text{Ҕ}(\text{A}, \text{A}\cup\text{B}) = \text{Ҕ}(\text{B}, \text{A}\cap\text{B})$
- **g)** $\text{Ҕ}(\text{A}, \overline{\text{A}}) = \text{Ҕ}(\overline{\text{A}}, \text{A})$

h) $\text{Ꝃ}(\overline{\text{₳}}, \overline{\text{฿}}) = \text{Ꝃ}(\text{₳}, \text{฿})$

i) $\text{Ꝃ}(\text{₳}, \overline{\text{฿}}) = \text{Ꝃ}(\overline{\text{₳}}, \text{฿})$

j) $\text{Ꝃ}(\overline{\text{₳}}, \overline{\text{฿}}) + \text{Ꝃ}(\overline{\text{₳}}, \text{฿}) = \text{Ꝃ}(\text{₳}, \text{฿}) + \text{Ꝃ}(\text{₳}, \overline{\text{฿}})$

2.4 A novel approach to the fuzzy MCDM problem

The components and the connections make up the two fundamental components of the network structure.

The elements that are grouped together and share a characteristic are called components. These include options, criteria, and the main purpose. Assume a set of k alternatives D= (D_1, D_2… D_k) and a set of l attributes E= (E_1, E_2… E_l) (Bozorg-Haddad et al, 2021; Rani et al., 2020).

The following FS’s describe the features of the alternative D_i in the matter of E specification:

$$D_i = \{< E_j, \upsilon_{ij} >, E_j \epsilon E\}. \qquad (8)$$

Where i ranges from 1 to k while j from 1 to l.

υ_{ij} is the accordance to which E_j is fulfilled by the alternative D_i.

Sorting through the options and selecting the best one is the aim of the decision-making task mentioned above.

3 APPLICATIONS OF DIVERGENCE MEASURE IN MADM

In today's multifaceted and shifting environment, making sound decisions is more vital than ever. Multi-attribute decision-making (MADM) is at the forefront of addresses that assist decision-makers assess and decide on the best selections based on a number conflicting criteria.

The successful application of MADM strategies is primarily dependent on decision-makers' capability to clearly and in statistical terms express their individual situations for decision-making. Decision-makers are entrusted with choosing the best option from a list of realistic alternatives, whose can often be affected by price, effectiveness, uncertainty, & customer preferences. As a result, the capacity to prioritize these aspects and build a list for preference becomes essential.

The concept of satisfaction is central to MADM, as it focuses on figuring out alternatives that provide the fullest achievement for the decision-maker's objectives. The literature indicates MADM is frequently used in an assortment of disciplines, including economics, management, and engineering, illustrating its versatility and widespread applicability. For example, in economic making decisions, MADM procedures allow stakeholders to assess investment opportunities by determining different financial indicators, while in management, they aid strategic planning through carrying out an organized assessment of competing priorities.

The procedure for solving MADM problem is as follows (Criado & Gachechiladze, 1997; Jain & Chhabra, 2014):

STEP I: Evaluate D^+ and D^- using:

$D^+ = \{<\upsilon_1{}^+>, <\upsilon_2{}^+>, \ldots, <\upsilon_l{}^+>\}$ and
$D^- = \{<\upsilon_1{}^->, <\upsilon_2{}^->, \ldots, <\upsilon_l{}^->\}$
Where for each j = 1, 2, 3… l.
$<\upsilon_j{}^+> = \{max_i\ \upsilon_{ij}\}$
$<\upsilon_j{}^-> = \{min_i\ \upsilon_{ij}\}$

STEP II: Calculate Ҕ (D^+, D_i) and Ҕ (D^-, D_i) using equation (7), we obtain (Chhabra & Chhabra, 2023; Jain & Chhabra, 2014):

$$Ҕ(D^+, D_i) = \frac{1}{2}\sum_{i=1}^{p}\left(\upsilon_{D^+}(z_i) - \upsilon_{D_i}(z_i)\right)^4\left[\frac{1}{\left(\upsilon_{D^+}(z_\iota) + \upsilon_{D_\iota}(z_\iota)\right)^3} + \frac{1}{\left(2 - \upsilon_{D^+}(z_\iota) - \upsilon_{D_\iota}(z_\iota)\right)^3}\right]. \quad (9)$$

And

$$Ҕ(D^-, D_i) = \frac{1}{2}\sum_{i=1}^{p}\left(\upsilon_{D^-}(z_i) - \upsilon_{D_i}(z_i)\right)^4\left[\frac{1}{\left(\upsilon_{D^-}(z_\iota) + \upsilon_{D_\iota}(z_\iota)\right)^3} + \frac{1}{\left(2 - \upsilon_{D^-}(z_\iota) - \upsilon_{D_\iota}(z_\iota)\right)^3}\right]. \quad (10)$$

STEP III: Compute the alternative D_i's relative fuzzy divergence measure Ҕ (D_i) in relation to D^+ and D^-, where:

$$Ҕ(D_i) = \frac{Ҕ(D^+, D_i)}{Ҕ(D^+, D_i) + Ҕ(D^-, D_i)}. \quad (11)$$

STEP IV: Sort your preferences in accordance with the divergence measure's estimated numerical values. The best option will be the one corresponding to the divergence measure's minimal value. The supplication of the suggested divergence measure in MADM, which provides a new fuzzy divergence measure, is demonstrated here supported by numerical example.

4 NUMERICAL EXAMPLE

Assuming that an investment firm, as described by Kahraman (2008), wishes to allocate a specific sum of money to the best of 5 options:

(1) D_1, a car company
(2) D_2, a food firm
(3) D_3, a computer company
(4) D_4, a weapons manufacturer and
(5) D_5, a television company.

The following four factors must be taken into consideration by the investing firm:
(1) E_1, the risk assessment;
(2) E_2, the examination of growth;
(3) E_3, the social-political effects study; and
(4) E_4, the surroundings effects study.

The decision-maker has created IFSs as the following five characteristic sets in order to evaluate the five potential alternatives $D_{i\,=}$ (one, 2... 5)

$D_1 = \{< E_1,\ .799 > , < E_2,\ .501 >, < E_3,\ .90 >, < E_4,\ .199 >\}$
$D_2 = \{< E_1,\ .210 > , < E_2,\ .699 >, < E_3,\ .611 >, < E_4,\ .619 >\}$
$D_3 = \{< E_1,\ .60 > , < E_2,\ .410 >, < E_3,\ .69 >, < E_4,\ .511 >\}$
$D_4 = \{< E_1,\ .510 > , < E_2,\ .30 >, < E_3,\ .40 >, < E_4,\ .720 >\}$
$D_5 = \{< E_1,\ .6001 > , < E_2,\ .50 >, < E_3,\ .501 >, < E_4,\ .70 >\}$

Calculation steps are as follows:

Table 1. The membership values of the surviving beings in relation to characteristic of contamination.

E_i / D_i	E_1	E_2	E_3	E_4
D_1	0.799	0.501	0.90	0.199
D_2	0.210	0.699	0.611	0.619
D_3	0.60	0.410	0.69	0.511
D_4	0.510	0.30	0.40	0.702
D_5	0.6001	0.50	0.501	0.70

STEP I: We obtain D^+ and D^- :

$D^+ = \{< E_1,\ .799 >, < E_2,\ .699 >, < E_3,\ .90 >, < E_4,\ .702 >\}$

And

$D^- = \{< E_1,\ .210 >, < E_2,\ .30 >, < E_3,\ .40 >, < E_4,\ .199 >\}$

STEP II: The calculated numerical values of Ꝁ (D^+, D_i) and Ꝁ (D^-, D_i) are shown in Tables 2 and 3:

Table 2. The calculated numerical values of Ꝁ (D+, Di).

$\text{Ꝁ}(D^+, D_i)$	$\text{Ꝁ}(D^+, D_i) = \frac{1}{2}\sum_{i=1}^{p}\left(\upsilon_{D^+}(z_i) - \upsilon_{D_i}(z_i)\right)^4\left[\frac{1}{\left(\upsilon_{D^+}(z_i) + \upsilon_{D_i}(z_i)\right)^3} + \frac{1}{\left(2 - \upsilon_{D^+}(z_i) - \upsilon_{D_i}(z_i)\right)^3}\right]$
Ꝁ (D^+, D_1)	0.068371
Ꝁ (D^+, D_2)	0.163369
Ꝁ (D^+, D_3)	0.027314
Ꝁ (D^+, D_4)	0.144583
Ꝁ (D^+, D_5)	0.069945

Table 3. The calculated numerical values of Ꝁ (D-, Di) .

$\text{Ꝁ}(D^-, D_i)$	$\text{Ꝁ}(D^-, D_i) = \frac{1}{2}\sum_{i=1}^{p}\left(\upsilon_{D^-}(z_i) - \upsilon_{D_i}(z_i)\right)^4\left[\frac{1}{\left(\upsilon_{D^-}(z_i) + \upsilon_{D_i}(z_i)\right)^3} + \frac{1}{\left(2 - \upsilon_{D^-}(z_i) - \upsilon_{D_i}(z_i)\right)^3}\right]$
Ꝁ (D^-, D_1)	0.236957
Ꝁ (D^-, D_2)	0.059607
Ꝁ (D^-, D_3)	0.054825
Ꝁ (D^-, D_4)	0.079997
Ꝁ (D^-, D_5)	0.100885

Table 4. The estimated values of Ꝁ (Di) for i=1, 2, 3,

Ꝁ(D_i)	Ꝁ$(D_i) = \frac{Ꝁ\,(D^+,D_i)}{Ꝁ\,(D^+,D_i)+Ꝁ\,(D^-,D_i)}$	Ranking
Ꝁ(D_1)	0.223926	1st
Ꝁ(D_2)	0.732673	5th
Ꝁ (D_3)	0.332536	2nd
Ꝁ(D_4)	0.643794	4th
Ꝁ(D_5)	0.409442	3rd

As seen in Table 4, The order of ranking is:

$D_1 > D_3 > D_5 > D_4 > D_2$

Hence, D_1 is the best alternative.

As a result, the provided numerical example shows that the divergence measure is an appropriate metric for resolving the MADM problem.

5 CONCLUSION

In conclusion, vagueness poses significant barriers in real-world decision-making, resulting in Atanassov's development of fuzzy sets to efficiently manage ambiguity. Fuzzy logic inclusion into multi-attribute decision-making (MADM) enables decision-makers to better handle imprecise information and subjective evaluations. This approach improves the evaluation of options across conflicting criteria, leading to better-informed decisions. For example, in healthcare, fuzzy MADM enables a harmonious mix of patient outcomes, expenses, and resource availability. Finally, implementing fuzzy addresses into MADM increases decision quality and stakeholder confidence, making it vital to navigating the complicated issues and uncertainties inherent in modern decision-making situations.

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The authors have contributed equally in this manuscript. All authors have read and accepted the published version of the manuscript.

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