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SOME APPLICATIONS OF FUZZY MEASURES WITH CODING THEORY

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Abstract: A Fuzzy measure have been seen to take part in developing various methods for the creation of fuzzy mean codeword lengths. This approach is taken by the current communication, which providing the application of fuzzy entropy measurements for the creation of novel fuzzy codeword lengths. Additionally, we want to provide more light on the problems of correspondence between weighted mean and possible weighted fuzzy entropy using fuzzy measures.

Keywords: Fuzzy entropy, Fuzzy mean codeword length, Weighted mean, Monotonic function.

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Algunas Aplicaciones de Medidas Difusas con la Teoría de Codificación

Resumen: Se ha visto que las medidas difusas participan en el desarrollo de varios métodos para la creación de longitudes de codecs difusas medias. Este enfoque es el que adopta la presente comunicación, que proporciona la aplicación de medidas difusas de entropía para la creación de nuevas longitudes difusas de codecs. Además, queremos aportar más luz sobre los problemas de correspondencia entre la media ponderada y la posible entropía difusa ponderada utilizando medidas difusas.

Palabras clave: Entropía difusa, longitud media difusa de la palabra clave, media ponderada, función monótona.

1 INTRODUCTION

Fuzziness and uncertainty are the basic nature of human thinking and many real-world objectives. Fuzziness is found in our decision, in our language and in the way of process information. The main objective of information is to remove uncertainty and fuzziness. In fact, we measure information supplied by the amount of probabilistic uncertainty removed in an experiment and the measure of uncertainty removed is also called as a measure of information, while measure of fuzziness is the measure of vagueness and ambiguity of uncertainties.

Fuzzy entropy is a concept that combines fuzzy set theory and information theory (Singh & Sharma, 2019), particularly in coding theory. Fuzzy entropy measures are mathematical tools used to quantify the uncertainty or randomness in a system where data might be imprecise, uncertain or fuzzy. They extend traditional entropy measures by incorporating the concept of fuzziness, which is particularly useful in dealing with real-world problems where information is not always precise. One of the many applications of fuzzy entropy measures in the literature will be to the problem of efficient coding of messages to be sent over a noiseless channel.

Fuzzy entropy can be applied to improve data compression algorithms. Traditional entropy-based methods, like Huffman coding or arithmetic coding, rely on precise probability distributions of symbols. However, in many real-world scenarios, the exact probabilities of symbols may not be well-defined or may vary over time. Fuzzy entropy allows for a more flexible representation of these probabilities, accounting for the uncertainty and variability in the data.

Fuzzy entropy can be used to develop adaptive coding schemes that adjust to the changing statistics of the data. By incorporating fuzzy sets into the entropy calculation, the coding algorithm can more effectively handle variations in the data stream, leading to better compression ratios. In image and video compression, where the data is often noisy and imprecise, fuzzy entropy can improve the performance of lossy compression algorithms like JPEG or MPEG. By considering the fuzziness in pixel values, the compression algorithm can achieve higher compression rates while maintaining acceptable quality. In communication systems, error correction codes are essential for ensuring data integrity over noisy channels. Fuzzy entropy measures can enhance the design of these codes by accounting for uncertainties in the error patterns. Traditional error correction techniques like Reed-Solomon or Turbo codes can be improved using fuzzy logic principles. Fuzzy entropy can be used to model the uncertainty in the received signal and enhance the decoding process (Germe Demamu et al., 2023) making it more robust to noise and errors. Fuzzy entropy can be used to model the characteristics of communication channels, especially those with uncertain or time-varying conditions. This modeling can lead to the development of more efficient and adaptive error correction schemes that perform better under varying channel conditions. For example: Imagine a scenario where you are transmitting data over a wireless network with varying signal quality. Traditional entropy-based methods might not efficiently compress the data due to the changing statistics of the signal. By applying fuzzy entropy, you can create a compression algorithm that adapts to these variations, achieving better compression ratios and reducing the overall data transmission time. Similarly, for error correction, a fuzzy entropy-based decoder can more effectively handle the uncertainties in the received data, leading to fewer errors and a more reliable communication system (Ouahada, & Ferreira, 2019). For data compression, fuzzy entropy-based algorithms compress data more effectively. Fuzzy entropy enhances cryptographic security. Fuzzy entropy analysis optimizes channel capacity.

Let's assume that a random variable K determines the messages to be conveyed and that each value of $K_k, k = 1, 2, \dots, \hbar$ must be denoted by a limited order of codes picked at random from the $O_1, O_2, \dots \dots \dots O_\rho$.

In discussing fuzzy coding theory, (Kraft, 1949) inequality plays a crucial $\hbar_k$ aspect. $\wp$ and provide the length of the code word related to this inequality as well as the size of the alphabet.

$$\sum_{k=1}^{\hbar} \wp^{-\hbar_k} \leq 1 , \tag{1}$$

We frequently encounter codes in communication theory that reduce the length of these particular fuzzy code words:

$$\aleph = \sum_{k=1}^{\hbar} (\Im_{t_k} + (1 - \Im_{t_k}))\hbar_k , \tag{2}$$

(Guiasu & Picard, 1971) determined the measure as the weighted mean codeword length by (Belis & Guiasu, 1968) fuzzy entropy:

$$\aleph(\omega) = \frac{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1 - \Im_{t_k}))\hbar_k \varpi_k}{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1 - \Im_{t_k}))\varpi_k} , \tag{3}$$

exponentiated mean was provided for the value = 1, and another exponentiated mean was provided for the value =.

$$\overline{\aleph}_\phi = \frac{1}{\phi - 1} log_\wp \left(\frac{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1 - \Im_{t_k}))^\phi \wp^{(\phi-1)\hbar_k}}{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1 - \Im_{t_k}))^\phi} \right) , \tag{4}$$

(Kapur, 1998) has well-researched a number of measures and their uses to coding theory. In coding theory, the topic of mistake correction is typically ignored in favour of maximizing the quantity of messages. So, given a specified limit on codeword lengths, we get the least value of a mean codeword length.

There has been extensive discussion of various measures and how they apply to coding theory by (Frumin et al., 2017; Hayashi, 2019; Joshi, & Kumar,2018; Kawan & Yüksel, 2018; Lee & Chung, 2018; Ouahada & Ferreira. 2019). A new approach of generalization of Renyi's entropy of parameter β has been discussed by Rakhi Gupta and Satish Kumar (Gupta & Kumar 2022). Surender Singh and Sonam Sharma introduce a generalized fuzzy entropy measure and demonstrate its effectiveness in Multiple Attribute Decision Making (MADM) (Singh & Sharma 2019). Gereme and Demamu updated properties of Hamming distance of binary fuzzy codes over fuzzy vector space (Germe et al. 2023). They also discussed binary fuzzy codes and some properties of Hamming distance of fuzzy codes (Guiasu, & Picard, 1971).

2 FUZZY MEASURES AND FUZZY CODING THEORY

Fuzzy entropy measures provide a powerful tool for optimizing coding and compression in communication systems by effectively handling uncertainties and imprecise information. This leads to more efficient data transmission, better compression ratios, and improved error correction capabilities, making it a valuable approach in the design of modern communication systems. In the example below, we show how fuzzy entropy values and fuzzy codeword lengths are related.

2.1 Fuzzy codeword lengths through fuzzy measures

In the context of communication systems, the concept of using fuzzy measures to determine codeword lengths can significantly enhance coding efficiency by better accommodating uncertainty and imprecise information. This approach involves utilizing fuzzy entropy measures to optimize the length of codewords, ensuring that the coding scheme is both robust and adaptable to varying data characteristics. Here, we create a few fuzzy exponentiated mean codeword lengths that are previously known in the coding theory literature.

Theorem 2.1: If $\sigma_1, \sigma_2, \ldots \sigma_n$,measurements of a FUDC.

$$\aleph_{\nu,\vartheta,\upsilon} \geq [z_\vartheta^\nu(\tau)]_\upsilon - \frac{\upsilon(1-\vartheta)-(1-\nu)}{\vartheta-\nu} log_\wp \sum_{k=1}^{\hbar} \wp^{-\hbar_k} , \tag{5}$$

Here,

$$\aleph_{\nu,\vartheta,\upsilon} = \frac{1}{\nu-\vartheta}[(\nu-1)\aleph^\nu - \nu(\vartheta-1)\aleph^\vartheta], \tag{6}$$

υ is constant, ν, ϑ are real parameters, $z_\vartheta^\nu(\tau)$ is Kapur’s (1967) fuzzy entropy and

$$\aleph^\nu = \frac{1}{\nu-1} log_\wp \left(\frac{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1-\Im_{t_k}))^\nu \wp^{-\hbar_k(1-\nu)}}{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1-\Im_{t_k}))^\nu} \right)$$
$$\aleph^\vartheta = \frac{1}{\vartheta-1} log_\wp \left(\frac{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1-\Im_{t_k}))^\vartheta \wp^{-\hbar_k(1-\vartheta)}}{\sum_{k=1}^{\hbar} (\Im_{t_k} + (1-\Im_{t_k}))^\vartheta} \right), \tag{7}$$

are Kapur’s fuzzy mean codeword lengths.

Proof. Due to (Kapur,1994) the following measure has occurred.

$$\mathrm{E}(t,s)=\frac{1}{\phi-\varphi}log_{\wp}\left(\frac{\sum_{k=1}^{\hbar}(\Im_{t_k}+(1-\Im_{t_k}))^{\nu}(\Im_{s_k}+(1-\Im_{s_k}))^{1-\nu}}{\left(\sum_{k=1}^{\hbar}(\Im_{t_k}+(1-\Im_{t_k}))^{\vartheta}(\Im_{s_k}+(1-\Im_{s_k}))^{1-\vartheta}\right)^{\upsilon}}\right), \nu\neq 1, \vartheta\neq 1, \nu,\vartheta>0, \upsilon>0\ , \tag{8}$$

$$\mathrm{E}(t,s)\geq 0, \text{ letting } (\Im_{s_k}+(1-\Im_{s_k}))=\frac{\wp^{-\hbar_k}}{\sum_{k=1}^{\hbar}\wp^{-\hbar_k}} \text{ then,} \tag{9}$$

$$\frac{1}{\nu-\vartheta}\left[(\nu-1)\overline{\aleph_{\nu}}-\upsilon(\vartheta-1)\overline{\aleph_{\vartheta}}\right]\geq\frac{1}{\vartheta-\nu}log_{\wp}\left(\frac{\sum_{k=1}^{\hbar}(\Im_{t_k}+(1-\Im_{t_k}))^{\nu}}{\left(\sum_{k=1}^{\hbar}(\Im_{t_k}+(1-\Im_{t_k}))^{\vartheta}\right)^{\upsilon}}\right)-\frac{\upsilon(1-\vartheta)-(1-\nu)}{\vartheta-\nu}log_{\wp}\sum_{k=1}^{\hbar}\wp^{-\hbar_k}, \tag{10}$$

$$\aleph_{\nu,\vartheta,\upsilon}\geq[z_{\vartheta}^{\nu}(t)]_{\upsilon}-\frac{\upsilon(1-\vartheta)-(1-\nu)}{\nu-\vartheta}log_{\wp}\sum_{k=1}^{\hbar}\wp^{-\hbar_k}\ , \tag{11}$$

Exceptional Points:

1. If $\upsilon=1$, (12)

i.e.

$$\aleph_{\nu,\vartheta}\geq z_{\vartheta}^{\nu}(t)-log_{\wp}\sum_{k=1}^{\hbar}\wp^{-\hbar_k}, \tag{13}$$

Here:

$$\aleph_{\nu,\vartheta}=\frac{1}{\nu-\vartheta}\left[(\nu-1)\aleph^{\nu}-(\vartheta-1)\aleph^{\vartheta}\right], \tag{14}$$

As of order ν and type ϑ, $\aleph_{\nu,\vartheta}$ is the exponentiated mean, and $z_{\vartheta}^{\nu}(t)$ is Kapur's fuzzy entropy (Kapur, 1986).

Now, Relation (13) is the lies between $z_{\vartheta}^{\nu}(t)$ and $z_{\vartheta}^{\nu}(t)$ + 1.

2. For $\upsilon=1$, ϑ =1, (10) (15)

i.e.

$$\aleph^{\nu}\geq\Im_{\nu}(t)-log_{\wp}\sum_{k=1}^{\hbar}\wp^{-\hbar_k}, \tag{16}$$

Here $\aleph^{\nu}$is of order ν and $\Im_{\nu}(t)$ is Renyi's fuzzy entropy (Renyi, 1961).

So,$\aleph^{\nu}$lower bound lies between $\Im_{\nu}(t)$ and $\Im_{\nu}(t)$ + 1

(ii) Calculating current fuzzy codeword lengths

We include fuzzy Shannon's (1948) and (Campbell, 1965) fuzzy code-word lengths here.

Theorem 2.2: If $\sigma_1,\sigma_2,...\sigma_n$, are the measurements of a fuzzy FUDC lengths, then.

$$-\frac{1+log_{\wp}\varpi}{log_{\wp}\varpi}log_{\wp}\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))^{\frac{1}{1+log_{\wp}\varpi}}\varpi^{\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{1+log_{\wp}\varpi}}\wp^{-\hbar_k\left(\frac{log_{\wp}\varpi}{1+log_{\wp}\varpi}\right)}\right)\geq -\frac{1}{log_{\wp}\varpi}log_{\wp}\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))\varpi^{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}\right), \quad (17)$$

Here, $\varpi>0,\varpi\neq 1$.

Proof. Holder's disparity is known to be caused by

$$\sum_{k}^{\hbar}a_kb_k\geq\left(\sum_{k}^{\hbar}(b_k)^t\right)^{\frac{1}{t}}\left(\sum_{k}^{\hbar}(b_k)^s\right)^{\frac{1}{s}}, \quad (18)$$

Substituting

$$b_k=(\Im_{t_k}+(1-\Im_{t_k}))^{-\frac{1}{log_{\wp}\varpi}}\varpi^{\frac{-log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{log_{\wp}\varpi}}, \quad (19)$$

$$b_k=(\Im_{t_k}+(1-\Im_{t_k}))^{\frac{1}{log_{\wp}\varpi}}\varpi^{\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{log_{\wp}\varpi}}\wp^{-\hbar_k},\frac{1}{t}=-\frac{1}{log_{\wp}\varpi},\frac{1}{s}=\frac{1+log_{\wp}\varpi}{log_{\wp}\varpi}, \quad (20)$$

From relation (18),

$$\sum_{k=1}^{\hbar}\wp^{-\hbar_k}\geq\left[\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))^{-\frac{1}{log_{\wp}\varpi}}\varpi^{-\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{log_{\wp}\varpi}}\right)^{-log_{\wp}\varpi}\right]^{-\frac{1}{1+log_{\wp}\varpi}}, \quad (21)$$

$$\left[\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))^{\frac{1}{log_{\wp}\varpi}}\varpi^{\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{log_{\wp}\varpi}}\wp^{-\mathrm{I}_k}\right)^{\left(\frac{log_{\wp}\varpi}{1+log_{\wp}\varpi}\right)}\right]^{\frac{1+log_{\wp}\varpi}{log_{\wp}\varpi}}, \quad (22)$$

i.e.,

$$\sum_{k=1}^{\hbar}\wp^{-\hbar_k}\geq\left[\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))\varpi^{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}\right)\right]^{\frac{-1}{log_{\wp}\varpi}} \left[\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))^{\frac{1}{1+log_{\wp}\varpi}}\varpi^{\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{1+log_{\wp}\varpi}}\wp^{-\hbar_k\left(\frac{log_{\wp}\varpi}{1+log_{\wp}\varpi}\right)}\right)\right]^{\frac{1+log_{\wp}\varpi}{log_{\wp}\varpi}}, \quad (23)$$

Or,

$$0\geq-\frac{1}{log_{\wp}\varpi}log_{\wp}\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))\varpi^{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}\right) +\frac{1+log_{\wp}\varpi}{log_{\wp}\varpi}log_{\wp}\sum_{k=1}^{\hbar}\left((\Im_{t_k}+(1-\Im_{t_k}))^{\frac{1}{1+log_{\wp}\varpi}}\varpi^{\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{1+log_{\wp}\varpi}}\wp^{-\hbar_k\left(\frac{log_{\wp}\varpi}{1+log_{\wp}\varpi}\right)}\right), \quad (24)$$

The relationship between fuzzy entropy and fuzzy non-mean codeword length is shown in equation (17).

Particular Cases:

Case-I: Adding $\varpi = \wp$,(17) results in

$$-2\, log_{\wp} \sum_{k=1}^{\hbar} \left((\Im_{t_k} + (1-\Im_{t_k}))^{\frac{1}{2}} \wp^{\frac{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{2}} \wp^{-\hbar_k\left(\frac{1}{2}\right)} \right) \geq -\, log_{\wp} \sum_{k=1}^{\hbar} \left((\Im_{t_k} + (1-\Im_{t_k})) \wp^{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))} \right), \tag{25}$$

$$-2\, log_{\wp} \sum_{k=1}^{\hbar} \left((\Im_{t_k} + (1-\Im_{t_k})) \wp^{-\hbar_k\left(\frac{1}{2}\right)} \right) \geq -\, log_{\wp} \sum_{k=1}^{\hbar} \left((\Im_{t_k} + (1-\Im_{t_k}))^2 \right), \tag{26}$$

This inequality shows the relationship between (Campbell, 1965) and (Renyi, 1961).

(ii) Relationship between Fuzzy Mean and Possible Fuzzy Entropy

The following weighted mean is first defined:

$$\aleph^{\nu}(\omega) = \frac{1}{\nu-1} log_{\wp} \left[\frac{\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k}+(1-\Im_{t_k}))^{\nu} \wp^{-\hbar_k(1-\nu)}}{\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k}+(1-\Im_{t_k}))^{\nu}} \right]; \nu > 1, \tag{27}$$

We noticed,

$$lm_{\nu\to 1} \aleph^{\nu}(\omega) = \left[\frac{\sum_{k=1}^{\hbar} \frac{\omega_k\left(\Im_{t_k}+\left(1-\Im_{t_k}\right)\right)\hbar_k + \left(\Im_{t_k}+\left(1-\Im_{t_k}\right)\right)}{log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))} - \sum_{k=1}^{\hbar} \omega_k\left(\Im_{t_k}+\left(1-\Im_{t_k}\right)\right) log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))}{\sum_{k=1}^{\hbar} \omega_k\left(\Im_{t_k}+\left(1-\Im_{t_k}\right)\right)} \right], \tag{28}$$

$$= \frac{\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k}+(1-\Im_{t_k}))\hbar_k}{\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k}+(1-\Im_{t_k}))}, \tag{29}$$

It is by (Guiasu, & Picard, 1971).

Fuzzy Universal Data Compression (FUDC) is a method that combines fuzzy set theory with universal data compression techniques to optimize the lengths of codewords. This approach leverages the principles of fuzzy entropy to account for the uncertainty and variability in the data, ultimately improving compression efficiency. FUDC involves assigning codeword lengths based on fuzzy probabilities, which are derived from the membership functions of fuzzy sets. This method enhances traditional data compression techniques by incorporating fuzziness, leading to more adaptable and robust compression. In below theorem, define new inequalities based on FUDC length.

Theorem 2.3: If $\sigma_1, \sigma_2, \ldots \sigma_n$, are the lengths of a FUDC lengths, then

$$\aleph^{\nu}(\omega) \geq z_{\nu}(t,\omega) - log_{\wp} \sum_{k=1}^{\hbar} \wp^{-\hbar_k} \;, \tag{30}$$

Here,

$$z_{\nu}(t,\omega) = \frac{1}{1-\nu} log_{\wp}\left(\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right) , \tag{31}$$

The weighted fuzzy mean is defined above, and $\aleph^{\nu}(\omega)$ is a potential indicator of weighted fuzzy entropy.

Proof: We utilize the possible weighted divergence provided by to demonstrate the previously mentioned theorem.

$$E_{\nu}(t,s;\omega) = \frac{1}{\nu-1}\left[tan^{-1}\left(\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} (\Im_{s_k} + (1-\Im_{s_k}))^{1-\nu}\right) - \frac{\pi}{4}\right], \nu > 1 \;, \tag{32}$$

It should be noticed that fuzzy measure (27) reduces to, when weights are ignored.

$$\mathrm{E}_{\nu}(t,\nu) = \frac{1}{\nu-1}\left[tan^{-1}\left(\sum_{k=1}^{\hbar} (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} (\Im_{s_k} + (1-\Im_{s_k}))^{1-\nu}\right) - \frac{\pi}{4}\right], \tag{33}$$

This represents the fuzzy Kapur (1994) measure.

Now, $\mathrm{E}_{\nu}(t,s;\omega) \geq 0$.

$$\Rightarrow \left[tan^{-1}\left(\sum_{k=1}^{\hbar} (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} (\Im_{s_k} + (1-\Im_{s_k}))^{1-\nu}\right)\right] \geq \frac{\pi}{4} \;, \tag{34}$$

Adding $(\Im_{s_k} + (1-\Im_{s_k})) = \frac{\wp^{-\hbar_k}}{\sum_{k=1}^{\hbar} \wp^{-\hbar_k}}$, in (33), (35)

$$tan^{-1}\left(\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} \left(\frac{\wp^{-\hbar_k}}{\sum_{k=1}^{\hbar} \wp^{-\hbar_k}}\right)^{1-\nu}\right) \geq \frac{\pi}{4}, \tag{36}$$

$$\Rightarrow \sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} \wp^{-\hbar_k(1-\nu)} \geq \left(\sum_{k=1}^{\hbar} \wp^{-\hbar_k}\right)^{1-\nu}, \tag{37}$$

$$- log_{\wp}\left(\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} \wp^{-\hbar_k(1-\nu)}\right) \leq (\nu-1)\, log_{\wp}\left(\sum_{k=1}^{\hbar} \wp^{-\hbar_k}\right), \tag{38}$$

Adding $log_{\wp}\left(\sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right)$, in (37) (39)

$$\aleph^{\nu}(\omega) = z_{\nu}(t,\omega) - log_{\wp}\left(\sum_{k=1}^{\hbar} \wp^{-\hbar_k}\right), \tag{40}$$

This establishes the theorem.

Theorem 2.4: If $\sigma_1, \sigma_2, \dots \sigma_n$,are the lengths of a FUDC lengths, then

$$z^{\nu}(t,\omega) \geq \aleph(\omega), \tag{41}$$

Here,

$$z^{\nu}(t,\omega) = \sum_{k=1}^{\hbar} \omega_k (\Im_{t_k} + (1-\Im_{t_k}))^{\nu} \left[1 - log_{\wp}(\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right]; \nu > 1, \tag{42}$$

Is weighted fuzzy entropy and $\aleph(\omega)$ is fuzzy weighted function.

Proof: (Gurdial & Pessoa, 1977) theorem is used to demonstrate the mentioned theorem.

$$\frac{\nu}{\nu-1} log_{\wp}\left[\sum_{k=1}^{\hbar}(\Im_{t_k} + (1-\Im_{t_k}))\left(\frac{\omega_k}{\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))}\right)^{\frac{1}{\nu}} \wp^{-\hbar_k \frac{\nu-1}{\nu}}\right] \geq \frac{1}{1-\nu} log_{\wp}\left(\frac{\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))^{\nu}}{\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))}\right), \tag{43}$$

Here,

$$\aleph_{\nu}(\omega) = \frac{\nu}{1-\nu} log_{\wp}\left(\sum_{k=1}^{\hbar}(\Im_{t_k} + (1-\Im_{t_k}))\left(\frac{\omega_k}{\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))}\right)^{\frac{1}{\nu}} \wp^{-\hbar_k \frac{\nu-1}{\nu}}\right), \tag{44}$$

is parametric weighted fuzzy code word length.

With equation (38),

$$\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k} + (1-\Im_{t_k}))^{\nu} \leq \sum_{k=1}^{\hbar}\omega_k(\Im_{t_k} + (1-\Im_{t_k}))\left[\sum_{k=1}^{\hbar}(\Im_{t_k} + (1-\Im_{t_k}))\left(\frac{\omega_k}{\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))}\right)^{\frac{1}{\nu}} \wp^{-\hbar_k \frac{\nu-1}{\nu}}\right]^{\nu}, \tag{45}$$

Substituting,

$$\sum_{k=1}^{\hbar} a_k = -\omega_k^{\frac{\nu}{1-\nu}}(\Im_{t_k} + (1-\Im_{t_k}))^{\frac{\nu^2}{1-\nu}}\left(log_{\wp}(\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right)^{\frac{\nu}{1-\nu}}\wp^{-\hbar_k}, \tag{46}$$

$$\sum_{k=1}^{\hbar} b_k = -\omega_k^{\frac{\nu}{1-\nu}}(\Im_{t_k} + (1-\Im_{t_k}))^{\frac{\nu^2}{1-\nu}}\left(log_{\wp}(\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right)^{\frac{\nu}{1-\nu}}, \text{ We get} \tag{47}$$

$$(\Im_{t_k} + (1-\Im_{t_k})) = 1-\nu, (\Im_{s_k} + (1-\Im_{s_k})) = \frac{\nu-1}{\nu}, \tag{48}$$

and by (Kraft, 1949) equality, $\sum_{k=1}^{\hbar}\wp^{-\hbar_k} = 1$

$$0 \geq \frac{1}{1-\nu} log_{\wp}\left[-\sum_{k=1}^{\hbar}(\omega_k)^{\nu}(\Im_{t_k} + (1-\Im_{t_k}))^{\nu^2}\left(log_{\wp}(\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right)^{\nu}\wp^{-\hbar_k(1-\nu)}\right] + \frac{\nu}{\nu-1} log_{\wp}\left[-\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\left(log_{\wp}(\Im_{t_k} + (1-\Im_{t_k}))^{\nu}\right)\right], \tag{49}$$

$$-\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))^{\nu}\left(log_{\wp}(\Im_{t_k}+\left(1-\Im_{t_k}\right))^{\nu}\right) \le \left[-\sum_{k=1}^{\hbar}(\omega_k)^{\nu}(\Im_{t_k}+(1-\Im_{t_k}))^{\nu^2}\left(log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))^{\nu}\right)^{\nu}\wp^{-\hbar_k(1-\nu)}\right]^{\frac{1}{\nu}}, \quad (50)$$

with (18) and (45), we get $z^{\nu}(t,\omega) \ge \aleph(\omega)$

Where,

$$\aleph(\omega) = \left(\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))\right)\left[\sum_{k=1}^{\hbar}(\Im_{t_k}+(1-\Im_{t_k}))\left(\frac{\omega_k}{\sum_{k=1}^{\hbar}\omega_k(\Im_{t_k}+(1-\Im_{t_k}))}\right)^{\frac{1}{\nu}}\wp^{-\hbar_k\frac{\nu-1}{\nu}}\right], \quad (51)$$

$$-\left[\left(\sum_{k=1}^{\hbar}(\omega_k)^{\nu}(\Im_{t_k}+(1-\Im_{t_k}))^{\nu^2}\right)\left(log_{\wp}(\Im_{t_k}+(1-\Im_{t_k}))^{\nu}\right)^{\nu}\wp^{-\hbar_k(1-\nu)}\right]^{\frac{1}{\nu}}, \quad (52)$$

Neither its monotonic increasing function nor any weighted fuzzy mean codeword length correspond to that.

3 CONCLUSION

This approach has the advantage of potentially establishing several new fuzzy entropy measurements using fuzzy coding theorems. In this paper we introduced a fuzzy entropy and corresponding code word length. It is shown that many new fuzzy coding theorems can be developed by considering both the existing and new fuzzy entropy measures. The work can further be extended for other fuzzy entropy measures.

AUTHORS' CONTRIBUTIONS

The authors have contributed equally to this manuscript. All authors have read and accepted the published version of the manuscript.

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DATA AVAILABILITY STATEMENT

The work did not use data from any external repository.

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The authors declare no conflict of interest.

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