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RESEARCH ARTICLE

A theoretical framework of inverse fuzzy soft expert set with application in decision-making

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ABSTRACT

Soft sets and their extensions emerge as a powerful tool for handling uncertainty in decision making applications dealing with only one expert. Decision-making involving opinions of more than one expert can be effectively modeled by soft expert sets (SES) and fuzzy soft expert set (FSES). Building upon these concepts this article presents the concept and operations of Inverse Fuzzy Soft Expert Set (inverse FSES) by developing an enhanced algorithmic approach. The Inverse FSES combines the principles of inverse fuzzy sets and SES to model complex decision-making scenarios involving multiple experts. Inverse FSES provides a mechanism to quantify, aggregate and interpret expert judgement with precision. The proposed algorithm is validated using sustainable supplier case study. Sensitivity analysis by changing weights of parameters and by adding credibility weights to experts is also conducted. This article combines the mathematical concept with practical applicability thereby opening a new avenue in decision-making.

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Un marco teórico del conjunto de expertos blando difuso inverso con aplicación en la toma de decisiones

RESUMEN

Los conjuntos suaves y sus extensiones se presentan como una herramienta poderosa para gestionar la incertidumbre en aplicaciones de toma de decisiones que involucran a un solo experto. La toma de decisiones que involucra las opiniones de más de un experto puede modelarse eficazmente mediante conjuntos de expertos suaves (SES) y conjuntos de expertos suaves difusos (FSES). Partiendo de estos conceptos, este artículo presenta el concepto y el funcionamiento del Conjunto de Expertos Suaves Difusos Inversos (FSES inverso) mediante el desarrollo de un enfoque algorítmico mejorado. El FSES inverso combina los principios de los conjuntos difusos inversos y los SES para modelar escenarios complejos de toma de decisiones que involucran a múltiples expertos. El FSES inverso proporciona un mecanismo para cuantificar, agregar e interpretar el juicio de los expertos con precisión. El algoritmo propuesto se valida mediante un estudio de caso de un proveedor sostenible. También se realiza un análisis de sensibilidad modificando los pesos de los parámetros y añadiendo pesos de credibilidad a los expertos. Este artículo combina el concepto matemático con la aplicabilidad práctica, abriendo así una nueva vía en la toma de decisiones.

Palabras clave: toma de decisiones; conjunto blando difuso inverso; conjunto blando experto; conjunto blando experto difuso

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1. Introduction

Uncertainty is a prevalent factor in numerous fields, including engineering, medical science, economics, and environmental science. Traditional methods often fall short when it comes to addressing these uncertainties. Hence many approaches like theory of probability, fuzzy set (FS) theory (Zadeh, 1965), rough sets (Pawlak, 1982) and soft sets (Molodtsov, 1999) came in picture. FS theory (Zadeh, 1965) provides a means of modeling uncertainty by allowing gradual membership of elements in a set lying between 0 and 1, rather than relying on binary membership as in classical sets. Introduction of fuzzy sets garnered significant attention as they can be widely applied in various áreas (Adlassnig, 1986). Molodtsov (Molodtsov, 1999) defined soft set (SS) theory which helped in conveniently dealing with uncertainty and vagueness as it does not require defining of membership function. SS theory was combined with other frameworks to develop hybrid models (Jahanvi et al., 2026; Jaleel et al., 2024; Maji et al., 2001) that offer greater adaptability in uncertain environments. Many researchers worked on developing theoretical foundation of SS theory (Aygün & Kamaci, 2019; Maji et al., 2003; Sezgin & Atagün, 2011; Sezgin & ÇAM, 2025) and its applications in áreas like medical diagnosis (Singh & Bhardwaj, 2024) and smart clothing (Jahanvi & Singh, 2026)

In 2016, conceptual shift occured when Çetkin et al. (Çetkin et al., 2016) gave the concept of inverse soft sets (ISS) and inverse fuzzy soft sets (IFSS). Unlike traditional soft sets where parameters are mapped to subsets of the universal set, inverse SS reverse this mapping direction. Here each element in the universal set is mapped to a subset of parameters. Khalil and Hassan (Khalil & Hassan, 2019) further proposed an algorithm using max-min and min-max decision of IFSS for a DM problem. Majority of the above listed models involve only a single expert. So basically, if one wants to include the opinion of several experts, then some operations need to be performed. This

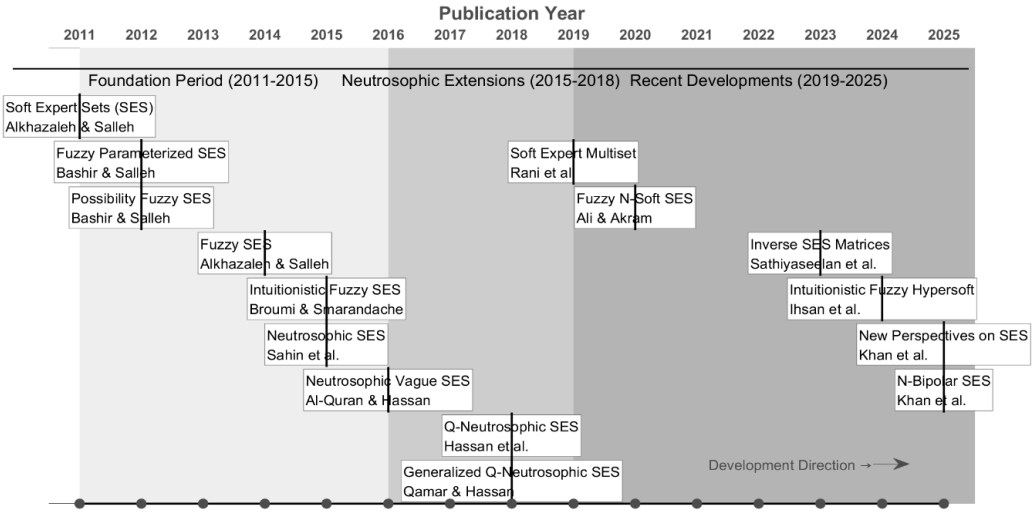


Figure 1. Evolution of soft expert set theory.

leads to an issue with the users specially with those who employ surveys for their research. The idea of soft expert set (SES) (Alkhazaleh & Salleh, 2011), which gives the user access to all experts' viewpoints in a single model without any actions, was established in an attempt to overcome this constraint. They also introduced the fuzzy SES (FSES) (Alkhazaleh & Salleh, 2014) that combines SES and FS theory which find applications in fields like prediction of diseases (Hassan et al., 2017), graph based MCDM (Şahin & Uluçay, 2024), prediction system (Liu et al., 2023) etc. SES theory is also integrated with other mathematical structures (Akram, Ali, Alcantud, et al., 2022; Ali & Akram, 2020; Chen et al., 2024; Khan et al., 2025) to increase the efficiency and utility. Akram et al. (Akram, Ali, Peng, et al., 2022) and Ali et al. (Ghous et al., 2021) presented spherical fuzzy N-SES and bipolar SES respectively. Evolution of soft expert set theory and its hybrid structures over the years is shown in the figure 1. Vijayabalaji et al. (Vijayabalaji et al., 2024) employed SES approach in rough FS for a MCDM case. The article (Liu, 2023) formulated fuzzy soft expert matrices and described different fuzzy soft expert matrices and their operations. Recently Sathiyaseelan (Sathiyaseelan et al., 2023) introduced inverse SES (ISES), symmetric matrices on ISES (SMISES), and its application.

So, this work introduces the idea of inverse fuzzy soft expert set which extends both inverse fuzzy soft sets and SES. The inverse FSES enables the integration of multiple expert opinions into a single model, providing an effective framework for decision-making (DM) under uncertainty.

The following are the primary contributions of this study:

1. The concept and structure of Inverse FSES is proposed with the help of illustrations.
2. Extended existing soft set and fuzzy set theory by integrating multiple expert opinions.
3. Provided a generalised framework for multi-expert DM.
4. The applicability of this concept is demonstrated through a decision making algorithm which is validated using a case study.
5. Performed the sensitivity analysis by adding expert credibility and changing parameter weights.

Remaining sections of this work are classified as follows. Second section firstly provides some definitions related to the background of the study and contains the main structure of the inverse FSES, and operations on inverse FSES including union, intersection, AND and OR. An application of inverse FSES using a case study is presented in section 3. Discussion and sensitivity analysis is given in section 4. The last section concludes the study with the limitations and future scope of the work done.

2. Theoretical framework

Firstly this section discusses some basic concepts required for the study. Consider the following notations for the preliminaries: U - universe set, $P(U)$ - power set of U, E - set of parameters, and $P(E)$ - power set of E.

Definition 2.1 (Zadeh, 1965) Let $\S \in U$. The FS G defined on U is a collection of ordered pairs,

$$G = \{(\S, \mu_G(\S)) | \S \in U\},$$

Here membership function (MF) $\mu_G(\S)$ maps each element of U to a membership value in $[0, 1]$.

Definition 2.2 (Molodtsov, 1999) A SS on U is defined as the ordered pairs

$$\delta = \{(e, F(e)) : e \in E, F(e) \in P(U)\},$$

where F is the mapping given by $F : E \to P(U)$.

Definition 2.3 (Çetkin et al., 2016) A pair (F, U) is called an inverse SS over E where F is the mapping denoted by $F : U \to P(E)$.

Definition 2.4 (Maji et al., 2001) Let I^U be the set of all FSs of U. A pair (F, E) is called a FSS over U, where $F : E \to I^U$ is the mapping F.

Definition 2.5 (Hassan et al., 2017) A pair $\left(\tilde{F}, U\right)$ is called an inverse FSS over E where $\tilde{F}$ is the mapping given by $\tilde{F} : U \to I^E$.

For definitions 2.6 and 2.7 given below, we consider M to be a set of expert agents, O be a set of opinions with two opinions 0 and 1 representing agree and disagree respectively, $Z = E \times M \times O$ and $A \subseteq Z$.

Definition 2.6 (Alkhazaleh & Salleh, 2011) A pair (F, A) is called a SES over U, where F represents mapping shown by $F : A \to P(U)$.

Definition 2.7 (Alkhazaleh & Salleh, 2014) A pair (F, A) is called a FSES over U, where F is a mapping given by $F : A \to I^U$ where I^U denotes the set of all fuzzy subsets of U.

Now the definition of Inverse FSES supported by an example is presented in this section. Further some operations on inverse FSES are defined.

Definition 2.8 Let U be universe, X denotes the parameters' set, M be a set of experts and O be a set of expert opinions $O = \{agree = 1, disagree = 0\}$. Then $\left(\tilde{\mathcal{F}}, U\right)$ is said to be an inverse fuzzy soft expert set where $\tilde{\mathcal{F}}$ is a mapping given by $\tilde{\mathcal{F}} : U \to I^X \times M \times O$ where I^X denotes the set of all fuzzy sets over X.

Example 2.1 Let's consider that a pregnant couple wants to select a maternity hospital with the help of the opinion of experts. Let $U = \{u_1, u_2, u_3\}$ represents three maternity hospitals, $X = \{d, c, a, ic, e\}$ is decision parameters' set whose elements denote "proximity to residence/distance", "cost", "24-hour availability of ob/gyn"," insurance coverage" and "maternity care experience".

Table 1. Representation of inverse FSES $(\tilde{\mathcal{F}}, U)$ including parameters d, c, a.

$(\tilde{\mathcal{F}}$,U)	(d,p, 1)	(d,p, 0)	(d,q, 1)	(d,q, 0)	(d,r, 1)	(d,r, 0)	(c,p, 1)	(c,p, 0)	(c,q, 1)	(c,q, 0)	(c,r, 1)	(c,r, 0)	(a,p,
u_1	0.2	0.1	0.6	0.2	0.2	0.2	0.1	0.2	0.1	0.1	0.3	0.2	0.4
u_2	0.7	0.1	0.3	0.6	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.5	0.4
u_3	0.2	0.2	0.3	0.2	0.4	0.2	0.4	0.8	0.2	0.7	0.3	0.4	0.4

Table 2. Representation of inverse FSES $(\tilde{\mathcal{F}}, U)$ including parameters $ic,\ e$.

$(\tilde{\mathcal{F}}$,U)	(ic,p, 1)	(ic,p, 0)	(ic,q, 1)	(ic,q, 0)	(ic,r, 1)	(ic,r, 0)	(e,p, 1)	(e,p, 0)	(e,q, 1)	(e,q, 0)	(e,r, 1)	(e,r, 0)
u_1	0.5	0.5	0.5	0.5	0.5	0.5	1	0.3	0.1	0.3	0.3	0.3
u_2	0.3	0.8	0.5	0.5	0.5	0.7	0.3	0.3	0.3	0.3	0.3	0.3
u_3	0.4	0.1	0.5	0.5	0.1	0.5	0.3	0.3	0.3	0.3	0.3	0.3

Let $M = \{p, q, r\}$ be a set of experts and O denotes the set of opinions where 1 stand for agree and 0 for disagree. Suppose that

$$\tilde{\mathcal{F}}(u_1) = \left\{\left(\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{1}\right), p, 1\right\}, \tilde{\mathcal{F}}(u_1) = \left\{\left(\frac{d}{0.6}, \frac{c}{0.2}, \frac{a}{0.1}, \frac{ic}{0.5}, \frac{e}{0.3}\right), q, 1\right\}, \tilde{\mathcal{F}}(u_1) = \left\{\left(\frac{d}{0.2}, \frac{c}{0.3},\right.\right.$$

The Inverse FSES $(\tilde{\mathcal{F}}, U)$ can be viewed as the following collection of approximations:

$$(\tilde{\mathcal{F}}, U) = \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{1}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.2}, \frac{a}{0.1}, \frac{ic}{0.5}, \frac{e}{0.1}\right\}, q, 1\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.3}, \frac{a}{0.4},\right.\right.$$

The tabular representation of Inverse FSES $(\tilde{\mathcal{F}}, U)$ is illustrated in Tables 1 and 2, where Table 1 comprises the first three parameters d, c, a and Table 2 comprises the remaining two parameters ic and e.

2.1. Operations on inverse FSES

Definition 2.9 For two Inverse FSESs $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, V)$ over $\mathfrak{O}$, $(\tilde{G}, V)$ is called inverse FSE subset of $(\tilde{\mathcal{F}}, U)$ if $V \subseteq U$ and $\forall \varepsilon \in U$, $\tilde{G}(\mathcal{E})$ is fuzzy subset of $\tilde{\mathcal{F}}(\mathcal{E})$.

This relationship is given by $(\tilde{G}, V) \subseteq (\tilde{\mathcal{F}}, V)$. Also, we can say that $(\tilde{\mathcal{F}}, U)$ is an inverse fuzzy soft expert superset of $(\tilde{G}, V)$.

Example 2.2 Consider Example 3.1. Let $U = (u_1, u_2, u_3)$ and $V = (u_1, u_2)$. Clearly $V \subseteq U$. Let $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, U)$ be defined as shown below:

$$(\tilde{\mathcal{F}}, U) =$$
$$\left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.1}, \frac{a}{0.7}, \frac{ic}{0.5}, \frac{e}{0.4}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.4}, \frac{c}{0.9}, \frac{a}{0.3}, \frac{ic}{0.5}, \frac{e}{0.6}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.3}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, r, 1\right), \left(($$
$$(\tilde{G}, V) =$$
$$\left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.2}, \frac{a}{0.1}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, r, 1\right), \left((u_2), \left\{\frac{d}{0.2}, \frac{c}{0.3}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 0\right), \left(($$

Therefore $(\tilde{G}, V) \subseteq (\tilde{\mathcal{F}}, V)$.

Definition 2.10 Two Inverse FSESs $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, U)$ over a parameter set which is common for both X are known as inverse fuzzy soft expert equal if both the sets are inverse fuzzy soft expert subsets of each other. It is denoted by $(\tilde{\mathcal{F}}, U) = (\tilde{G}, U)$.

Definition 2.11 An agree Inverse FSES $(\tilde{\mathcal{F}}, U)_1$ over X is an inverse fuzzy soft expert subset of $(\tilde{\mathcal{F}}, U)$ defined as follows:

$$\left(\tilde{\mathcal{F}}, U\right)_1 = \tilde{\mathcal{F}}_1(a) : a \in I^X \times M \times 1 ,$$

Definition 2.12 A disagree Inverse FSES $\left(\tilde{\mathcal{F}}, U\right)_0$ over X is an inverse fuzzy soft expert subset of $\left(\tilde{\mathcal{F}}, U\right)$ defined as follows:

$$\left(\tilde{\mathcal{F}}, U\right)_0 = \tilde{\mathcal{F}}_0(a) : a \in I^X \times M \times 0. ,$$

Example 2.3 Consider Example 3.1. Then the agree Inverse FSES $\left(\tilde{\mathcal{F}}, U\right)_1$ and the disagree Inverse FSES $\left(\tilde{\mathcal{F}}, U\right)_0$ is as follows:

$$\left(\tilde{\mathcal{F}}, U\right) = \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{1}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.2}, \frac{a}{0.1}, \frac{ic}{0.5}, \frac{e}{0.1}\right\}, q, 1\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.3}, \frac{a}{0.4}, \frac{}{0}\right.\right.$$

$$\left(\tilde{\mathcal{F}}, U\right) = \left((u_1), \left\{\frac{d}{0.1}, \frac{c}{0.2}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, q, 0\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.2}, \frac{a}{0.4}\right.\right.$$

Definition 2.13 The complement of Inverse FSES $\left(\tilde{\mathcal{F}}, U\right)$ is denoted by $\left(\tilde{\mathcal{F}}, U\right)^c$ and is defined by $\left(\tilde{\mathcal{F}}, U\right)^c = \left(\tilde{\mathcal{F}}^c, U'\right)$ where $\tilde{\mathcal{F}}^c : U \to I^X \times M \times O$ is a mapping given by $\tilde{\mathcal{F}}^c a = c\left(\tilde{\mathcal{F}}(a)\right), \forall a \in U$ where c is a fuzzy complement.

Example 2.4 *Consider Example 3.1. With the help of fuzzy complement, we get*

$$\left(\tilde{\mathcal{F}}, U\right) = \left((u_1), \left\{\frac{d}{0.8}, \frac{c}{0.9}, \frac{a}{0.6}, \frac{ic}{0.5}, \frac{e}{0}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.4}, \frac{c}{0.8}, \frac{a}{0.9}, \frac{ic}{0.5}, \frac{e}{0.9}\right\}, q, 1\right), \left((u_1), \left\{\frac{d}{0.8}, \frac{c}{0.7}, \frac{a}{0.6}, \frac{}{0}\right.\right.$$

Definition 2.14 The union of two Inverse FSESs $\left(\tilde{\mathcal{F}}, U\right)$ and $\left(\tilde{G}, V\right)$ denoted by $\left(\tilde{\mathcal{F}}, U\right) \cup \left(\tilde{G}, V\right)$, is the Inverse FSES $\left(\tilde{\mathcal{H}}, W\right)$ where $W = U \cup V$, and $\forall \varepsilon \in W$,

$$\tilde{\mathcal{H}}(\varepsilon) = \begin{cases} \tilde{F}(\varepsilon), & if\ \varepsilon \in U - V, \\ \tilde{G}(\varepsilon), & if\ \varepsilon \in V - U, \\ s(\tilde{\mathcal{F}}(\varepsilon), \tilde{G}(\varepsilon)) & if\ \varepsilon \in U \cap V, \end{cases} ,$$

where s is an s-norm.

Example 2.5 *Consider Example 3.2. Let $U = (u_1, u_2, u_3)$ and $V = (u_1, u_2)$.*
Suppose $\left(\tilde{\mathcal{F}}, U\right)$ and $\left(\tilde{G}, V\right)$ are two Inverse FSESs over $\mathfrak{V}$ such that

$$\left(\tilde{\mathcal{F}}, U\right) =$$
$$\left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.1}, \frac{a}{0.7}, \frac{ic}{0.5}, \frac{e}{0.4}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.4}, \frac{c}{0.9}, \frac{a}{0.3}, \frac{ic}{0.5}, \frac{e}{0.6}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.3}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, r, 1\right), \left((\right.$$
$$\text{and } \left(\tilde{G}, V\right) =$$
$$\left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.2}, \frac{a}{0.1}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, r, 1\right), \left((u_2), \left\{\frac{d}{0.2}, \frac{c}{0.3}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{3}\right\}, p, 0\right), \left((u_2\right.$$

By using the basic fuzzy union (maximum) we have $\left(\tilde{\mathcal{F}}, U\right) \cup \left(\tilde{G}, V\right) = \left(\tilde{\mathcal{H}}, W\right)$ where

$$\left(\tilde{\mathcal{H}}, W\right) = \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.1}, \frac{a}{0.7}, \frac{ic}{0.5}, \frac{e}{0.4}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.4}, \frac{c}{0.9}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.6}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.3}, \frac{a}{0.4}\right.\right.$$

Definition 2.15 The intersection of two Inverse FSESs $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, V)$ over $\mathfrak{V}$ denoted by $(\tilde{\mathcal{F}}, U) \cap (\tilde{G}, V)$, is the Inverse FSES $(\tilde{\mathcal{H}}, W)$ where $W = U \cup V$, and $\forall \varepsilon \in W$,

$$\tilde{\mathcal{H}}(\epsilon) = \begin{cases} \tilde{F}(\epsilon), & if\ \epsilon \in U - V, \\ \tilde{G}(\epsilon), & if\ \epsilon \in V - U, \\ t(\tilde{\mathcal{F}}(\epsilon), \tilde{G}(\epsilon)) & if\ \epsilon \in U \cap V, \end{cases}$$

where t is an t-norm.

Example 2.6 *Consider Example 3.5. By taking the use of fuzzy intersection, we have* $(\tilde{\mathcal{F}}, U) \cap (\tilde{G}, V) = (\tilde{\mathcal{H}}, W)$ *where*

$$(\tilde{\mathcal{H}}, W) = \left((u_1), \left\{\frac{d}{0.6}, \frac{c}{0.1}, \frac{a}{0.7}, \frac{ic}{0.5}, \frac{e}{0.4}\right\}, p, 1\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.3}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 0\right), \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.2}, \frac{a}{0.1}\right.\right.$$

Definition 2.16 If $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, V)$ are two inverse fuzzy soft expert sets over $\mathfrak{V}$ then $(\tilde{\mathcal{F}}, U)$ *AND* $(\tilde{G}, V)$ denoted by $(\tilde{\mathcal{F}}, U) \wedge (\tilde{G}, V)$, is defined by

$$(\tilde{\mathcal{F}}, U) \wedge (\tilde{G}, V) = (\tilde{\mathcal{H}}, U \times V)\ ,$$

such that $\tilde{\mathcal{H}}(a, b) = t\left(\tilde{\mathcal{F}}(a), \tilde{G}(b)\right), \forall (a, b) \in U \times V$, where t is a t-norm.

Example 2.7 Let $U = (u_1,\ u_2,\ u_3)$, $V = (u_1, u_2)$, $\mathcal{M} = (p, q)$, and $\mathcal{O} = (0, 1)$.

Suppose $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, V)$ are two inverse FSESs over $\mathfrak{V}$ such that

$$(\tilde{\mathcal{F}}, U) = \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 1\right), \ \left((u_2), \left\{\frac{d}{0.6}, \frac{c}{0.2}, \frac{a}{0.1}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 1\right),$$

$$(\tilde{G}, V) = \left((u_1), \left\{\frac{d}{0.2}, \frac{c}{0.7}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.1}\right\}, p, 1\right), \ \left((u_2), \left\{\frac{d}{0.8}, \frac{c}{0.2}, \frac{a}{0.7}, \frac{ic}{0.5}, \frac{e}{0.6}\right\}, p, 1\right),$$

By the use of basic fuzzy intersection we have $(\tilde{\mathcal{F}}, U) \wedge (\tilde{G}, V) = (\tilde{\mathcal{H}}, U \times V)$ where

$$(\tilde{\mathcal{H}}, U \times V) = \left((u_1, u_1), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.1}\right\}, p, 1\right), \left((u_1, u_2), \left\{\frac{d}{0.2}, \frac{c}{0.1}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 1\right), \left((u_2, u_1\right.\right.$$

Definition 2.17 If $(\tilde{\mathcal{F}}, U)$ and $(\tilde{G}, V)$ are two inverse fuzzy soft expert sets over $\mathfrak{V}$ then $(\tilde{\mathcal{F}}, U)$ *OR* $(\tilde{G}, V)$ denoted by $(\tilde{\mathcal{F}}, U) \vee (\tilde{G}, V)$, is defined by

$$(\tilde{\mathcal{F}}, U) \vee (\tilde{G}, V) = (\tilde{\mathcal{H}}, U \times V)\ ,$$

such that $\tilde{\mathcal{H}}(a, b) = s\left(\tilde{\mathcal{F}}(a), \tilde{G}(b)\right), \forall (a, b) \in U \times V$, where s is a s-norm.

Example 2.8 Consider Example 3.7. By the use of basic fuzzy union we get $(\tilde{\mathcal{F}}, U) \vee (\tilde{G}, V) = (\tilde{\mathcal{H}}, U \times V)$ where

$$(\tilde{\mathcal{H}}, U \times V) = \left((u_1, u_1), \left\{\frac{d}{0.2}, \frac{c}{0.7}, \frac{a}{0.4}, \frac{ic}{0.5}, \frac{e}{0.3}\right\}, p, 1\right), \left((u_1, u_2), \left\{\frac{d}{0.8}, \frac{c}{0.2}, \frac{a}{0.7}, \frac{ic}{0.5}, \frac{e}{0.6}\right\}, p, 1\right), \left((u_2, u_1\right.\right.$$

3. Methodology

This section discusses a real-life application of Inverse FSES theory in a DM problem. An inverse FSES based DM algorithm is proposed.

3.1. Decision-Making Algorithm

Step 1: Construct and input the Inverse FSES.

Step 2: Find the agree Inverse FSES and disagree Inverse FSES.

Step 3: Define weights for each parameter based on importance.

Step 4: Normalize the membership values formula:

$$x'_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}} ,$$

Step 5: Compute weighted aggregation: for agree Inverse FSES:

$$S_i^+ = \sum_{j=1}^{4} w_j \times x_{ij}^+ ,$$

For disagree Inverse FSES:

$$S_i^- = \sum_{j=1}^{4} w_j \times x_{ij}^- ,$$

Step 6: Compute net score and ranking net score:

$$NS_i = S_i^+ - S_i^- ,$$

Rank alternatives in descending order of net score. The highest score indicates the optimal choice. Figure 2 shows the flowchart for the decisión making algorithm.

3.2. Case study: sustainable supplier selection

Sustainable supplier selection is a critical strategic decision in supply chain management, balancing economic, environmental, and social criteria under uncertainty.

3.2.1. Problem formulation

Let the universe set be $U = \{s_1, s_2, s_3, s_4, s_5\}$ representing five candidate suppliers for a manufacturing firm seeking to enhance supply chain sustainability. Let $E = \{e_1, e_2, e_3\}$ be a set of experts where:

- e_1: Procurement Manager
- e_2: Sustainability Officer
- e_3: Operations Analyst

Let $O = \{1, 0\}$ be the set of opinions where 1 denotes ”agree” and 0 denotes ”disagree”.

Let $P = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ be the set of evaluation parameters defined as:

Applying the proposed dm algorithm to our selected database:

Step 1: Construct and input inverse FSES for the given data.

Step 2: Organize the Agree inverse FSES and Disagree inverse FSES into two separate tables. Table 4 and 5 represent agree inverse FSES, and Table 6 and 7 represent disagree inverse FSES.

Step 3: Assign weights for each parameter based on importance as: $w_1 = 0.20,\ w_2 = 0.18,\ w_3 = 0.15,\ w_4 = 0.17,\ w_5 = 0.15,\ w_6 = 0.15$

Step 4: Apply normalization to membership degrees to ensure comparability using the formula:

Table 3. Sustainability evaluation parameters.

Parameter	Description	Category
p_1	Environmental Management System (ISO 14001)	Environmental
p_2	Carbon Emission Reduction Capability	Environmental
p_3	Cost Competitiveness	Economic
p_4	Quality Performance (Defect Rate)	Economic
p_5	Green Innovation Capacity	Social/Innovation
p_6	Ethical Labor Practices	Social

Table 4. Agree inverse FSES $(\tilde{F}, Z)_1$ for parameters p_1, p_2, p_3.

Supp-lier	$(\mathbf{p}_1,\mathbf{e}_1)$	$(\mathbf{p}_1,\mathbf{e}_2)$	$(\mathbf{p}_1,\mathbf{e}_3)$	$(\mathbf{p}_2,\mathbf{e}_1)$	$(\mathbf{p}_2,\mathbf{e}_2)$	$(\mathbf{p}_2,\mathbf{e}_3)$	$(\mathbf{p}_3,\mathbf{e}_1)$	$(\mathbf{p}_3,\mathbf{e}_2)$	$(\mathbf{p}_3,\mathbf{e}_3)$
s_1	0.82	0.78	0.75	0.71	0.68	0.73	0.65	0.62	0.68
s_2	0.65	0.70	0.68	0.82	0.85	0.80	0.78	0.75	0.72
s_3	0.88	0.92	0.85	0.90	0.88	0.86	0.55	0.58	0.52
s_4	0.45	0.48	0.50	0.42	0.45	0.48	0.88	0.85	0.90
s_5	0.72	0.75	0.70	0.78	0.80	0.76	0.70	0.72	0.68

$$x'_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}},$$

Step 5: Compute weighted aggregation.

Step 6: The difference of agree sum and disagree sum gives the net score. Rank suppliersz in descending order of net score. The highest score indicates the best supplier. Table 8 presents the aggregated scores and final ranking.

The given algorithm based on inverse FSES framework can also be applied in other decision-making scenarios to provide the optimal choice.

Supplier s_3 is identified as the optimal sustainable supplier with the highest net score $S_3 = 0.2425$.

4. Discussion

For evaluating the reliability of the findings obtained, a sensitivity analysis is conducted. It helps determine how changes in the input parameters affect the final decision or outcome. This analysis provides insights into the stability of the model and identifies potential vulnerabilities to variations or uncertainties in the data. It also helps identify potential areas for improvement or refinement of the model.

Table 5. Agree inverse FSES $(\tilde{F}, Z)_1$ for parameters p_4, p_5, p_6.

Supp-lier	$(\mathbf{p}_4,\mathbf{e}_1)$	$(\mathbf{p}_4,\mathbf{e}_2)$	$(\mathbf{p}_4,\mathbf{e}_3)$	$(\mathbf{p}_5,\mathbf{e}_1)$	$(\mathbf{p}_5,\mathbf{e}_2)$	$(\mathbf{p}_5,\mathbf{e}_3)$	$(\mathbf{p}_6,\mathbf{e}_1)$	$(\mathbf{p}_6,\mathbf{e}_2)$	$(\mathbf{p}_6,\mathbf{e}_3)$
s_1	0.70	0.72	0.68	0.85	0.88	0.82	0.78	0.82	0.75
s_2	0.75	0.78	0.72	0.68	0.72	0.65	0.82	0.85	0.80
s_3	0.82	0.85	0.80	0.92	0.95	0.88	0.88	0.90	0.85
s_4	0.68	0.65	0.70	0.38	0.42	0.35	0.55	0.58	0.52
s_5	0.78	0.80	0.75	0.75	0.78	0.72	0.72	0.75	0.70

Table 6. Disagree inverse FSES $(\tilde{F}, Z)_0$ for parameters p_1, p_2, p_3.

Supp-lier	(p_1,e_1)	(p_1,e_2)	(p_1,e_3)	(p_2,e_1)	(p_2,e_2)	(p_2,e_3)	(p_3,e_1)	(p_3,e_2)
s_1	0.18	0.22	0.25	0.29	0.32	0.27	0.35	0.38
s_2	0.35	0.30	0.32	0.18	0.15	0.20	0.22	0.25
s_3	0.12	0.08	0.15	0.10	0.12	0.14	0.45	0.42
s_4	0.55	0.52	0.50	0.58	0.55	0.52	0.12	0.15
s_5	0.28	0.25	0.30	0.22	0.20	0.24	0.30	0.28

Table 7. Disagree inverse FSES $(\tilde{F}, Z)_0$ for parameters p_4, p_5, p_6.

Supp-lier	(p_4,e_1)	(p_4,e_2)	(p_4,e_3)	(p_5,e_1)	(p_5,e_2)	(p_5,e_3)	(p_6,e_1)	(p_6,e_2)	$(p_6,e_3,)$
s_1	0.30	0.28	0.32	0.15	0.12	0.18	0.22	0.18	0.25
s_2	0.25	0.22	0.28	0.32	0.28	0.35	0.18	0.15	0.20
s_3	0.18	0.15	0.20	0.08	0.05	0.12	0.12	0.10	0.15
s_4	0.32	0.35	0.30	0.62	0.58	0.65	0.45	0.42	0.48
s_5	0.22	0.20	0.25	0.25	0.22	0.28	0.28	0.25	0.30

4.1. Changing parameter weights

A crucial aspect of sensitivity analysis is adjusting the weights assigned to different parameters which can significantly influence the outcome. This is because the weights reflect the relative importance of each parameter in the decision-making process. In the drug selection case study different weights were assigned to the parameters based on their importance. The weights were assigned as follows:

$w_1 = 0.20,\ w_2 = 0.18,\ w_3 = 0.15,\ w_4 = 0.17,\ w_5 = 0.15,\ w_6 = 0.15.$

We consider three new sets of weights. "p_1 and p_2 focused" and "p_3 and p_4 focused," which emphasize the importance of environmental and economic category, respectively. Similarly "p_5 and p_6 focused" to test the scenario when social category is highlighted.

Set 1: $w_1 = 0.30, w_2 = 0.30, w_3 = 0.10, w_4 = 0.10, w_5 = 0.10$

Set 2: $w_1 = 0.10, w_2 = 0.10, w_3 = 0.30, w_4 = 0.30, w_5 = 0.10$

Set 3: $w_1 = 0.10, w_2 = 0.10, w_3 = 0.10, w_4 = 0.30, w_5 = 0.30$

We systematically apply these different weight sets to our model to observe how the outcomes vary. Table 9 represents the ranking after applying new weight sets. Figure 3 represents the combined graph of all the four cases.

The analysis reveals sensitivity of model when there is shift in the importance of these parameters. All other factors are constant while changing the weights to isolate the impact of the weight adjustments. This process helps identify the most critical parameters and understand the degree to which the results are sensitive to changes in their weights. This information is valuable for making informed decisions and understanding the robustness of the analysis.

Table 8. Aggregated scores and final ranking.

Supplier	A_i (Agree Score)	D_i (Disagree Score)	S_i (Net Score)	Rank
s_1	0.4522	0.3819	0.0703	3
s_2	0.4545	0.3686	0.0860	2
s_3	0.5008	0.2582	0.2425	1
s_4	0.3424	0.6420	−0.2996	5
s_5	0.4508	0.3810	0.0698	4

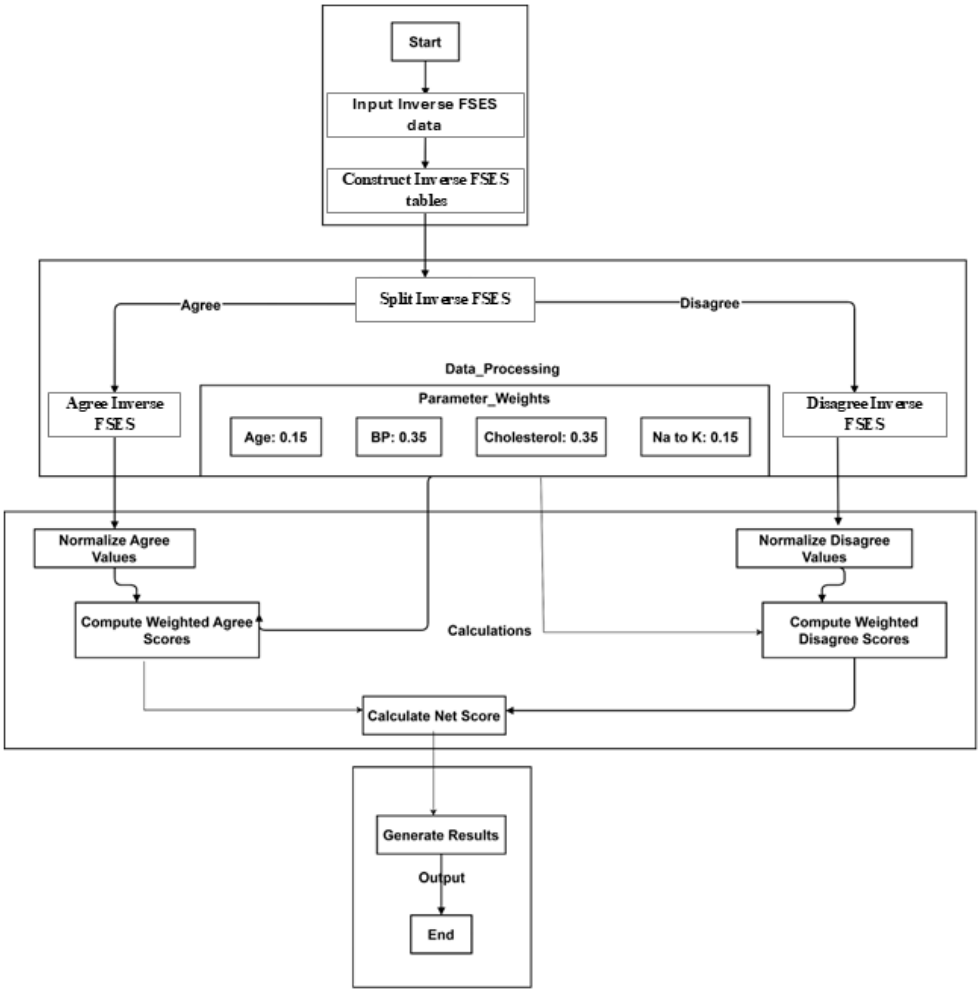


Figure 2. Inverse FSES algorithm flowchart.

Table 9. Supplier rankings after varying parameter weights.

Supplier	Base Case	Environmental	Economic	Social
s_1	3	4	4	2
s_2	2	2	2	3
s_3	1	1	1	1
s_4	5	5	5	5
s_5	4	3	3	4

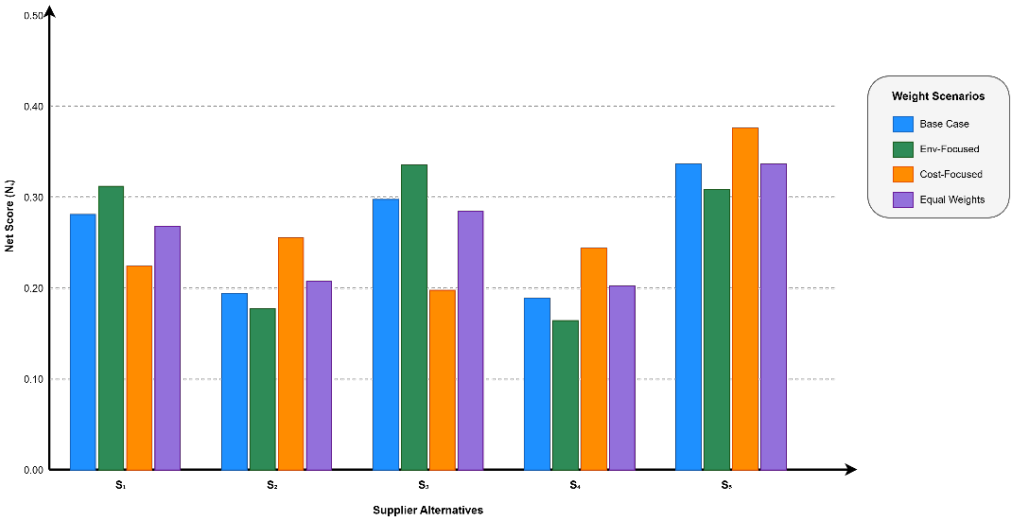


Figure 3. Sensitivity analysis of different parameter weight sets.

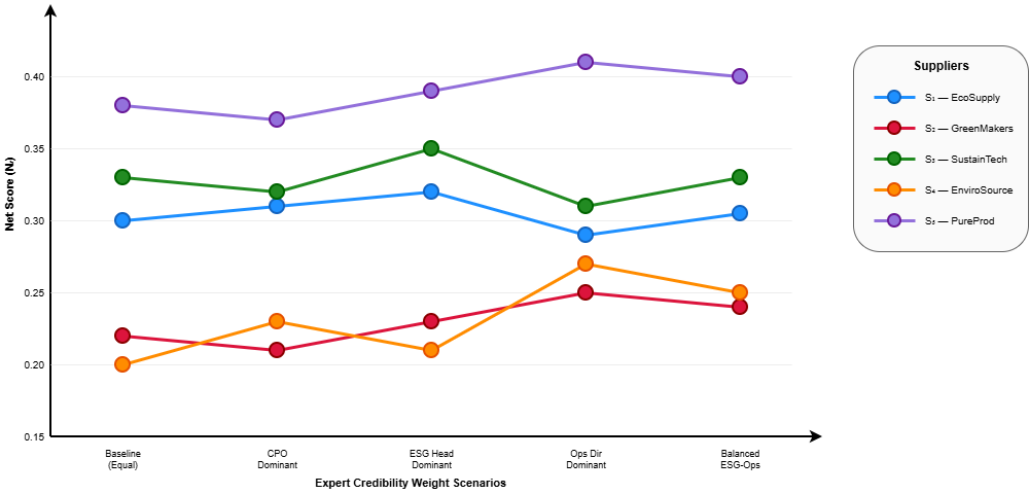


Figure 4. Sensitivity analysis of adding expert credibility weights.

4.2. Adding expert credibility weights

Inverse FSES considers multiple experts and in many situations, not all experts hold the same level of knowledge or experience. They can be weighted differently based on their level of expertise, experience and credibility. The results' sensitivity when changes are made in expert credibility is evaluated by adding weights to the experts and observing the impact on the final decision or net score. Originally no weights were assigned to the experts. Suppose the following weights are added to the experts as four cases:

Equal: $w_1 = 0.333,\ w_2 = 0.333,\ w_3 = 0.333.$

Procurement focused: $w_1 = 0.50,\ w_2 = 0.25,\ w_3 = 0.25$

Sustainibility focused: $w_1 = 0.25,\ w_2 = 0.50,\ w_3 = 0.25$

Table 10. Ranking stability under expert credibility variations.

Supplier	Equal	Procurement	Sustainability	Operations
s_3	1	1	1	1
s_5	2	2	2	2
s_1	3	3	4	3
s_2	4	4	3	4
s_4	5	5	5	5

Operations focused: $w_1 = 0.25,\ \ w_2 = 0.25,\ \ w_3 = 0.50$

Table 10 shows ranking stability under expert credibility variations. Figure 4 shows the impact of weight addition to experts. The ranking of supplier s_3 as the optimal choice remains stable across all expert credibility scenarios, demonstrating the robustness of the Inverse FSES framework. This ensures that the opinions of more credible experts have a greater influence on the final decision.

5. Conclusions

This article proposes the concept and operations of Inverse FSES which aims to offer a more comprehensive and efficient approach in dealing with expert opinions and uncertainties in complex systems. The inverse FSES framework through a real-world decision-making example involving sustainable supplier selection has been illustrated and an algorithm has been developed. Further sensitivity analysis is performed to evaluate the reliability of the results.

The only limitation for this method is that the computational complexity might increase with increase in the number of experts and parameters. However diverse case studies may be done to validate the inverse FSES approach.

In future, mathematical foundations of inverse FSES could be refined like taking the expert opinion as multivalued using fuzzy membership values and additional applications in different fields could be explored. Also, inverse FSES could be combined with matrices and inverse fuzzy soft expert matrices (IFSEM) can be developed to broaden the approach to decision making problems.

CRediT authorship contribution statement

Jahanvi: conceptualization, methodology, literature review, writing - original draft preparation, formal analysis. Rashmi Singh: supervision, validation, methodology, writing - reviewing and editing.

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Data availability statement

The data used for this study is included in the manuscript.

Declaration of competing interest

The authors declare no competing interests.

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