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Comparative analysis of the global innovation index combining the graph visualization and topological data analysis approaches

Análisis comparativo del índice global de innovación combinando los enfoques de visualización de grafos y análisis topológico de datos

Research article

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Rolando Ismael Yépez*
Instituto Superior Universitario Cotacachi, Cotacachi, Ecuador.
E-mail: iyepez@intitutocotacachi.edu.ec
Orcid: https://orcid.org/0000-0002-8127-629X

Maricela Fernanda Ormaza
Pontificia Universidad Católica del Ecuador, Ibarra, Ecuador.
E-mail: mformaza@pucesi.edu.ec
Orcid: https://orcid.org/0000-0002-6777-4567

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Abstract

This study analyzes the Global Innovation Index (GII) of the 100 most innovative countries in 2022 and 2023, applying the Fruchterman-Reingold algorithm to obtain a spatial distribution of the data and utilizing persistent homology with Vietoris-Rips complexes at three scales ($\varepsilon = 0.3$, 1.0, and 1.5) to form connected components or structures. The results reveal evolutionary patterns in the global innovation ecosystem. With $\varepsilon = 0.3$, connected components increase from 13 to 14 between 2022 and 2023, reflecting fragmentation that captures heterogeneity in innovation levels, with innovation islands such as Switzerland, United States, and Sweden appearing isolated from developing economies. At $\varepsilon = 1.5$, complete unification into a single connected component is observed, revealing an underlying continuity in the global innovation spectrum. This methodology complements traditional approaches by revealing structural transitions and topological distances between countries, providing a foundation for strategic interventions that could reduce persistent inequalities between innovation leaders and followers.

Keywords: Global Innovation Index, topological data analysis, innovation, persistent homology.

Resumen

Este estudio analiza el Índice Global de Innovación (GII) de los 100 países más innovadores en 2022 y 2023, aplicando el algoritmo Fruchterman-Reingold para obtener una distribución espacial de los datos y utilizando homología persistente con complejos de Vietoris-Rips a tres escalas ($\varepsilon = 0.3$, 1.0 y 1.5) para formar componentes o estructuras conectadas. Los resultados revelan patrones evolutivos en el ecosistema global de innovación. Con $\varepsilon = 0.3$, los componentes conectados aumentan de 13 a 14 entre 2022 y 2023, reflejando una fragmentación que captura la heterogeneidad en los niveles de innovación, con islas de innovación como Suiza, Estados Unidos y Suecia apareciendo aisladas de economías en desarrollo. A $\varepsilon = 1.5$, se observa una unificación completa en un único componente conectado, revelando una continuidad subyacente en el espectro innovador global. Esta metodología complementa los enfoques tradicionales al revelar transiciones estructurales y distancias topológicas entre países, proporcionando una base para intervenciones estratégicas que podrían reducir las persistentes desigualdades entre líderes y seguidores en innovación.

Palabras clave: Índice Global de Innovación, Análisis Topológico de Datos, innovación, homología persistente.

1. Introduction

Innovation is now a key element in driving economic growth and enhancing global competitiveness among countries. The Global Innovation Index (GII) annually assesses the performance of innovation ecosystems within major economies and examines current global innovation trends (Nasir & Zhang, 2024). Understanding the evolution of global innovation and its dynamics over time is fundamental for developing effective policies and supporting the strengthening of robust innovation ecosystems (Atsu & Adams, 2023).

The GII, published by the World Intellectual Property Organization (WIPO) in collaboration with INSEAD and Cornell University, provides a comprehensive analysis of countries' innovation capacities and outcomes. This index is built from a variety of indicators covering areas such as institutions, human capital and research, infrastructure, market development, and innovation development (Schiuma & Santarsiero, 2023). Through a multivariate evaluation approach, the index captures the complexity of the innovation process, enabling the identification of strengths and areas for improvement within the innovation ecosystem, and serving as a solid foundation for policy and strategy formulation (Borodin et al., 2021; Cottrell et al., 2024; Cui & Diwu, 2024). The indicators are grouped into two main sub-indices: the Input Sub-Index, which analyzes resources and investments in innovation, and the Output Sub-Index, which evaluates the outcomes of innovative activities (Hamdouch & Nyseth, 2023).

The analysis of these indices has traditionally been conducted using conventional statistical methods. However, in today's era of big data and advanced analytics, new tools have emerged that allow for a deeper and more multidimensional examination of the data. One such tool is Topological Data Analysis (TDA), which has proven to be a powerful method for uncovering hidden patterns and underlying structures in complex datasets (Carlsson, 2009; Cao et al., 2024; Skaf & Laubenbacher, 2022; Smith et al., 2021; Giunti et al., 2024).

In this context, the main objective of this research is to apply a methodology that combines graph visualization techniques and TDA to conduct a comparative analysis of the GII among the 100 most innovative countries in 2022 and 2023. This approach not only overcomes the limitations of traditional methods but also offers a new perspective for analyzing the evolution of global innovation over a specific time period. By focusing on this two-year window, we capture the most recent transition in the global innovation landscape following significant global events, allowing for the identification of immediate structural shifts and emerging patterns in the relative positioning of countries. This temporal resolution provides a timely assessment of how innovation ecosystems respond to contemporary global challenges while remaining computationally tractable for the complex topological analyses employed in this study. Furthermore, this focused timeframe enables policymakers to identify actionable insights based on current rather than historical innovation dynamics, particularly regarding the formation and dissolution of innovation clusters observed during this critical period.

The methodology is based on the use of the Fruchterman-Reingold algorithm, which is effective for creating spatially distributed graphs, as noted by Gajdoš et al. (2016). This is followed by the application of persistent homology analysis. This approach enables the visualization and examination of the complex relationships between the components of the GII, as well as the interactions and changes between the years 2022 and 2023, in a way that is not achievable with traditional statistical methods (Hansen et al., 2020; Jewell, 2022).

Topological Data Analysis (TDA) encompasses a set of tools designed for the visualization, exploration, and analysis of data based on topological principles, a branch of mathematics that studies abstract concepts related to shape and connectivity. Topology allows for the examination of data independently of the chosen coordinates, reducing sensitivity to the selection of the metric (Smith et al., 2021). Additionally, it extends graph theory techniques to high-dimensional spaces and is robust against noise (Skaf & Laubenbacher, 2022).

One of the approaches associated with TDA is the application of persistent homology to point clouds representing data. This involves transforming each cloud into a geometric object through a filtration process. During the filtration, the topology is measured at each stage, and topological features, such as holes, that appear and disappear are recorded (Smith et al., 2021). These features are represented in a persistence diagram, where the birth and death values are noted (Petelin et al., 2024).

The specific objectives of this study are: to apply the combined methodology of graph visualization and TDA to the GII data of the 100 most innovative countries in 2022 and 2023; to identify and measure the persistent topological features in these data at different scales; and to perform a comparative analysis between both years to gain a deeper understanding of the evolution of countries' innovation capacities.

It is expected that the results of this study will provide valuable insights into the dynamics of global innovation and how it has evolved in the short term. Furthermore, this work aims to demonstrate the value of advanced analytical techniques in examining globally relevant economic indices, opening new avenues for research in the fields of innovation and economic development. The use of TDA in this context could offer a novel perspective on how different dimensions of innovation interact and evolve over time, which could be particularly useful for policymakers and researchers in the field of global innovation.

2. Methodology

First, the data is extracted and initially processed, sourced from the GII sub-indices for the years 2022 and 2023.

Then, preprocessing is carried out using R (version 4.3.3) in the RStudio environment (version 2024.04.2+764). During this stage, missing values are identified and addressed.

Next, the graph visualization is performed. Undirected graphs (V,E) are constructed for each sub index using the igraph library (version 3.11.0). To achieve an optimal spatial distribution of the vértices V (countries) in the Euclidean plane, the Fruchterman-Reingold algorithm is applied, which generates a layout $L: V \rightarrow \mathbb{R}^{2.}$ This presentation produces coordinate matrices $M \in \mathbb{R}^{nx2}$, where n is the number of countries.

The next step involves Topological Data Analysis (TDA). For this, persistent homology is implemented using Vietoris-Rips complexes. A Vietoris-Rips complex $VR(X,\in)$ is defined as the abstract simplicial complex whose $k-$símplices correspond to subsets of $k+1$ points in X with pairwise maximum distance $\leq \in$. Using the TDA library (version 1.9.1), a filtration $\{VR(X,\in)\}_{\in \geq 0}$ is constructed for $\in \in [0,6]$. The persistent homology groups $H_k(VR(X,\in)\})$para $k=0,1$ are calculated, quantifying the number of connected components and one-dimensional cycles, respectively.

The results obtained are represented using persistence diagrams $D_k=\{(b_i,d_i)\}$, where b_i and d_i denote the birth and death values of each i-th topological feature of dimension k. Vietoris-Rips complexes are constructed and visualized for three specific scales: $\in=0.3$, $\in=0.9$ y $\in=1.5$ allowing detailed observation of emerging structures at different levels of resolution. For each scale $\in_i$, he Betti numbers $\beta_0(VR(X,\in i))$, are calculated, quantifying the connected components.

Additionally, a color scheme $C:\mathbb{R} \rightarrow [0,1]^3$ is implemented to represent the levels of innovation, facilitating the visual interpretation of the results. Finally, a comparative analysis is performed between the GII, based on the visual interpretation of the persistence diagrams and the connected components at different scales. The similarities and differences in the topological structures identified for each year are analyzed and the results are interpreted in the context of innovation theory and the findings from the 2022 and 2023 GII reports.

3. Results and discussion

3.1 Data and descriptive statistics

Figures 1 and 2 illustrate the innovation levels of the countries ranked among the top 100 according to the GII for the years 2022 and 2023, respectively.

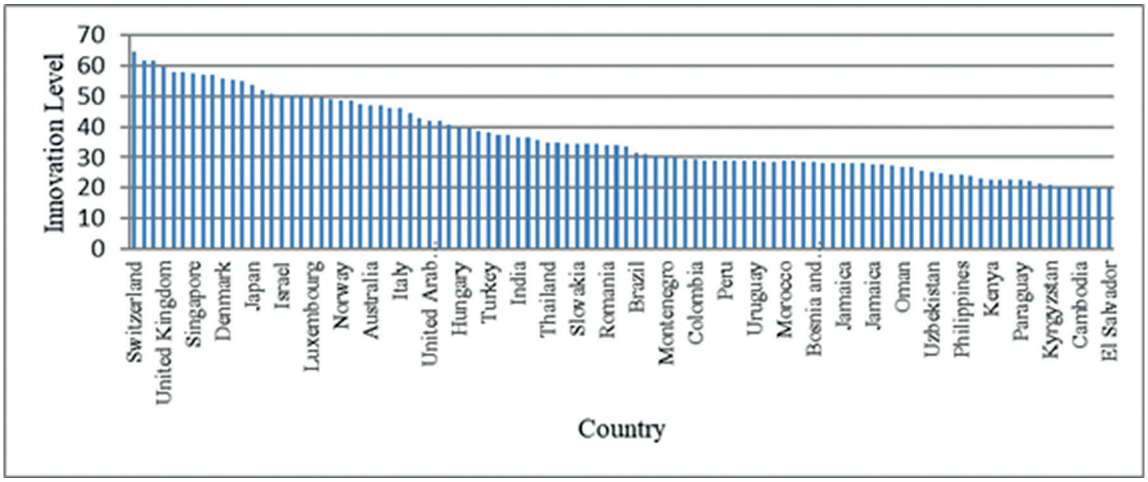

Figure 1. Innovation levels by country according to GII 2022.

The analysis of the GII 2022 reveals an innovation landscape primarily led by European nations, with Switzerland, the United States, and Sweden occupying the top three positions, followed by the United Kingdom and the Netherlands. The top 10 is completed by the Republic of Korea, Singapore, Germany, Finland, and Denmark. Emerging economies such as China (11th) and India (40th) stand out in the mid-range positions. In Latin America, Chile leads the region (50th), followed by Brazil (52nd) and Mexico (56th). The lowest ranks are mostly occupied by developing nations, including El Salvador (100th), Senegal (99th), Ecuador (98th), Cambodia (97th), and Namibia (96th).

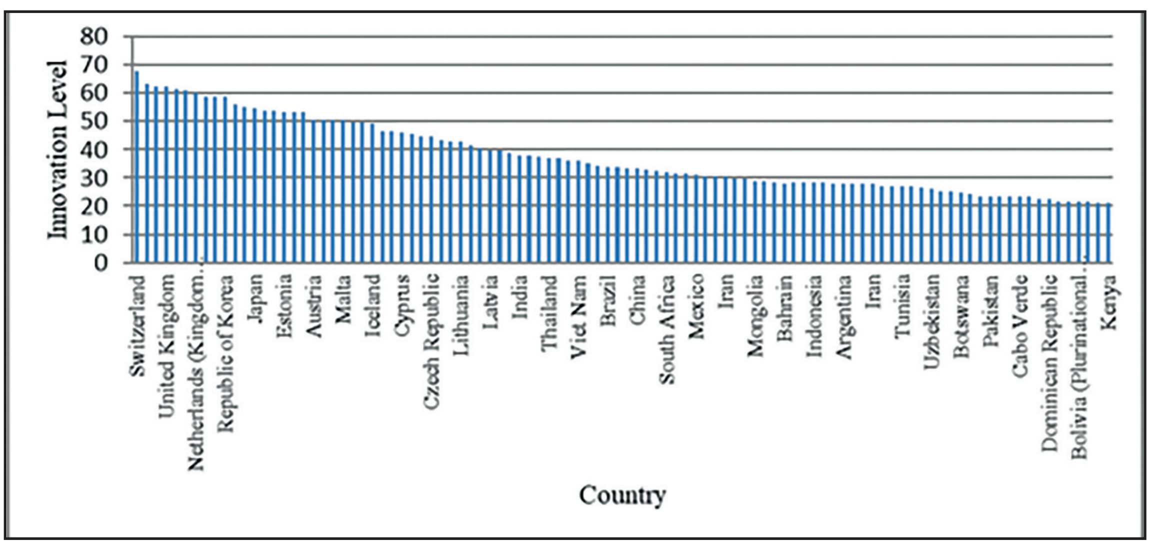

Figure 2. Innovation levels by country according to GII 2023.

The analysis of the GII 2023 reveals an innovation landscape primarily led by European nations, with Switzerland, Sweden, and the United States occupying the top three positions, followed by the United Kingdom and Singapore. The top 10 is completed by Finland, the Netherlands, Germany, Denmark, and the Republic of Korea. Emerging economies such as China (12th) and India (40th) stand out in the mid-range positions. In Latin America, Brazil leads the region (49th), followed by Chile (50th) and Mexico (58th). The lower positions of the top 100 are mainly occupied by developing countries such as Kenya (100th), the Philippines (99th), Paraguay (98th), Bolivia (97th), and Namibia (96th).

Below is a comparative table showing the descriptive statistics for GII in the years 2022 and 2023. Table 1 includes measures of central tendency and dispersion, such as the minimum, first quartile, median, mean, third quartile, maximum, and standard deviation for both years.

Table 1. Comparative descriptive statistics for the GII in 2022 and 2023.

Statistic	GII 2022	GII 2023
Minimum	19.90	21.24
1st Quartile	27.85	27.77
Median	33.70	33.34
Mean	36.35	37.58
3rd Quartile	46.38	47.23
Maximum	64.60	67.59
Standard Deviation	11.98	12.67

Overall, the GII values in 2023 show an increase compared to 2022 in terms of the mean, third quartile, and maximum, suggesting an overall improvement in global innovation over the past year. The minimum has also increased slightly, indicating that even the country with the lowest index has shown improvement. The median and first quartile, although showing slight variations, reflect stability in the central positions of the distribution.

The standard deviation has increased from 11.98 in 2022 to 12.67 in 2023, suggesting that the variability in the innovation index has grown, indicating a greater disparity among countries in terms of their innovative capacity.

In summary, although the GII values in 2023 have shown an increase at both the upper and lower ends of the distribution, as well as in the central values, the higher standard deviation indicates that the gap between countries with the highest and lowest indices has grown. This suggests that, while innovation indices have improved on average, there has also been an increase in inequality between leading countries and laggards in terms of innovative capacity.

3.2 Spatial distribution of the GII 2022 and 2023

Figures 3 and 4 display the graphs visualizing the spatial distribution of the 100 most innovative countries in the world according to the GII for 2022 and 2023, respectively. Both graphs were constructed using the Fruchterman-Reingold algorithm. The application of this algorithm has enabled a balanced and clear arrangement of the nodes in the graphs, facilitating the interpretation of the distribution of countries according to this index.

Figure 3. GII 2022 of the 100 most innovative countries, with spatial distribution based on the Fruchterman-Reingold algorithm.

Figure 4. GII 2023 of the 100 most innovative countries, with spatial distribution based on the Fruchterman-Reingold algorithm.

Each node in the graphs represents a country, and the spatial distribution reflects the similarities and differences between the countries based on their scores in the respective subindices. The connections between the nodes indicate proximities. From this spatial distribution of the data, a matrix with the coordinates of each point (country) was extracted, which served as input for applying the techniques of Topological Data Analysis (TDA).

3.3 Persistent homology based on Vietoris-Rips complexes

The persistence diagrams presented in Figures 5 and 6 provide a topological view of the underlying structures in the GII for the years 2022 and 2023, respectively, for the 100 most innovative countries. The 0-dimensional components are represented by yellow points, while the 1-dimensional cycles are represented by blue points.

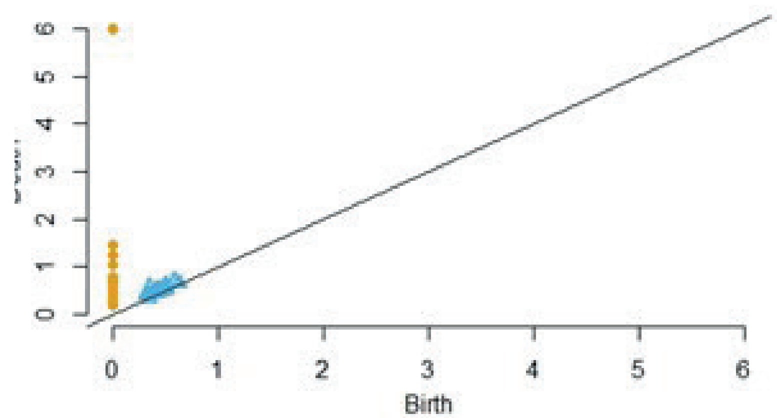

Figure 5. Persistence diagram of the GII
for the year 2022 of the 100 most innovative countries.

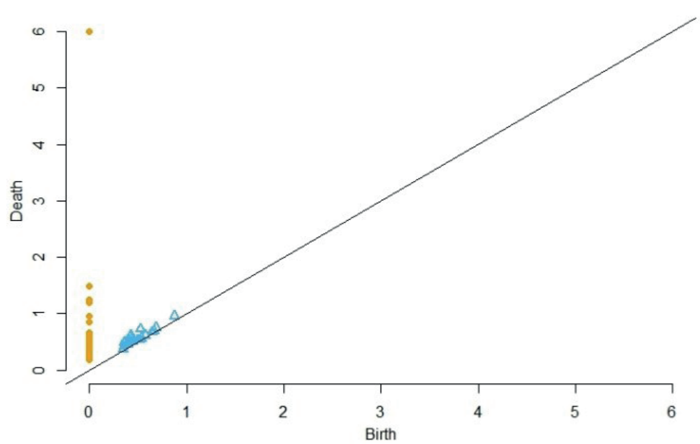

Figure 6. Persistence diagram of the GII for the year 2023 of the 100 most innovative countries.

When analyzing the persistence diagrams for the GII of the 100 most innovative countries in 2022 and 2023, significant characteristics are observed.

The distribution of points in the diagrams shows stability in the topological structure of the GII between the two years. The 0-dimensional components, represented by yellow points, indicate the presence of multiple groups of countries with persistent innovation characteristics. The 1-dimensional cycles, shown by blue points, suggest the existence of circular relationships between groups of countries. In 2023, the points appear more compact, and there is a slight reduction in long-persistence cycles compared to 2022. The presence of outlier points with high persistence may indicate leading innovative countries. Overall, the diagrams reflect a complex and stable topological structure in the global innovation space, with subtle changes between the two years.

3.4 Multi-scale connected components (ε = 0.3, 1, and 1.5) for the GII 2022 and 2023 using persistent homology with Vietoris-Rips complexes

Multi-scale connected components with ε = 0.3 for the GII 2022 and 2023

Figures 7 and 8 show the topological structures captured with Vietoris-Rips complexes with ε = 0.3 for the GII of the 100 most innovative countries in 2022 and 2023, respectively.

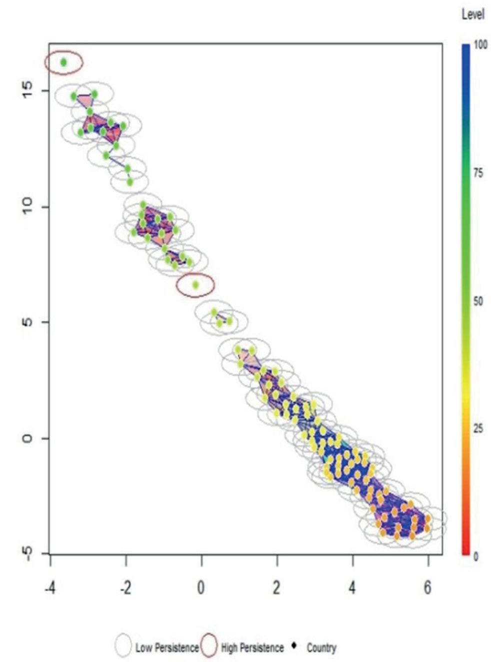

Figure 7. Topological structures captured with Vietoris-Rips complexes on the spatial distribution of the GII 2022 for the 100 most innovative countries (ε = 0.3).

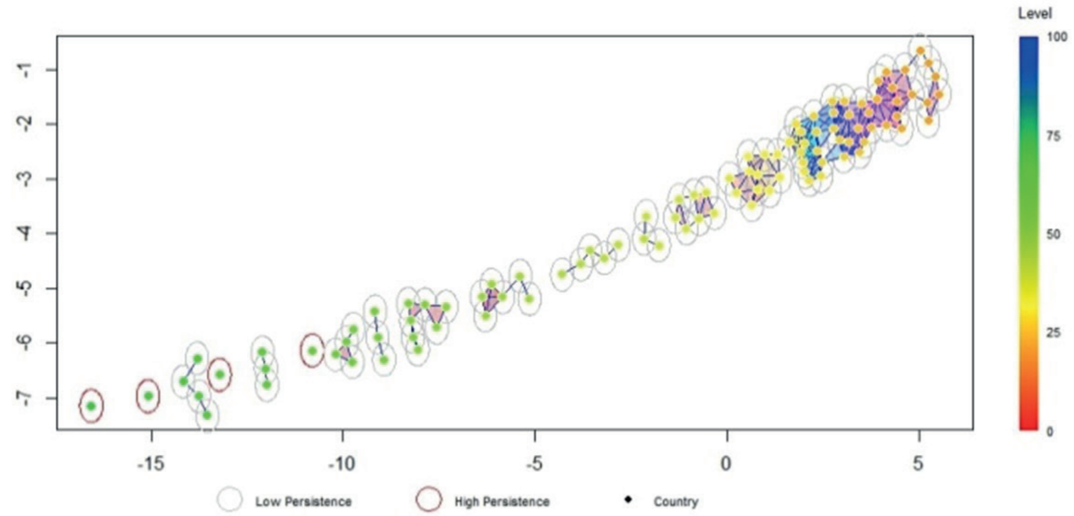

Figure 8. Topological structures captured with Vietoris-Rips complexes on the spatial distribution of the GII 2023 for the 100 most innovative countries (ε = 0.3).

In 2022, 13 connected components were observed in the innovation network. The principal component, located in the lower right quadrant, encompassed nations with GII values approximately between 20 and 40 (orange to yellow spectrum), predominantly representing developing and emerging economies from Latin America, Africa, and parts of Asia, including El Salvador (100), Ecuador (98), and Cambodia (97).

Eight smaller components (containing 2-9 countries each) were identified distributed from the central region toward the upper left quadrant, with GII values ranging between 40 and 65 (light green spectrum), representing upper-middle and high-income economies such as Italy (28), Czech Republic (30), and Estonia (18). Additionally, four isolated components were detected: one centrally positioned near the principal component, and the others distributed across the point cloud, with one at the extremity exhibiting a relatively high value marked in green, corresponding to Switzerland (1), the innovation leader.

For 2023, the number of connected components increased to 14. The principal component shifted to the upper right quadrant, encompassing countries with GII values between 23 and 45 (orange-yellow spectrum), including economies such as Kenya (100), Philippines (99), and Azerbaijan (89).

Nine smaller components (containing 2-9 countries each) were observed distributed from the central region toward the lower left quadrant, with GII values between 45 and 68 (light green spectrum), representing medium-high innovation economies such as Spain (29), Italy (26), and Australia (24). The four isolated components concentrated in the lower left quadrant in light green, corresponding to the highest innovation economies, with two positioned at the edge of the point cloud with relatively high values, including Switzerland (1), Sweden (2), and the United States (3).

At this ε level, the persistence of regional and economically similar clusters was demonstrated, as well as the presence of innovation leaders that remain isolated from the rest due to their exceptional performance.

Multiscale Connected Components with ε = 1 for the GII 2022 and 2023

Figures 9 and 10 show the topological structures captured with Vietoris-Rips complexes with ε = 1 from the GII for 2022 and 2023 for the 100 most innovative countries, respectively.

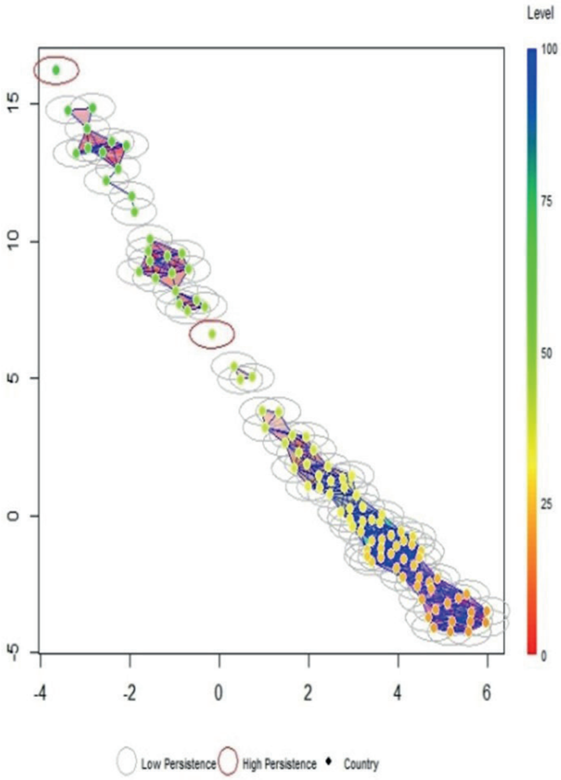

Figure 9. Topological structures captured with Vietoris-Rips complexes on the spatial distribution of the GII 2022 for the 100 most innovative countries (ε = 1).}

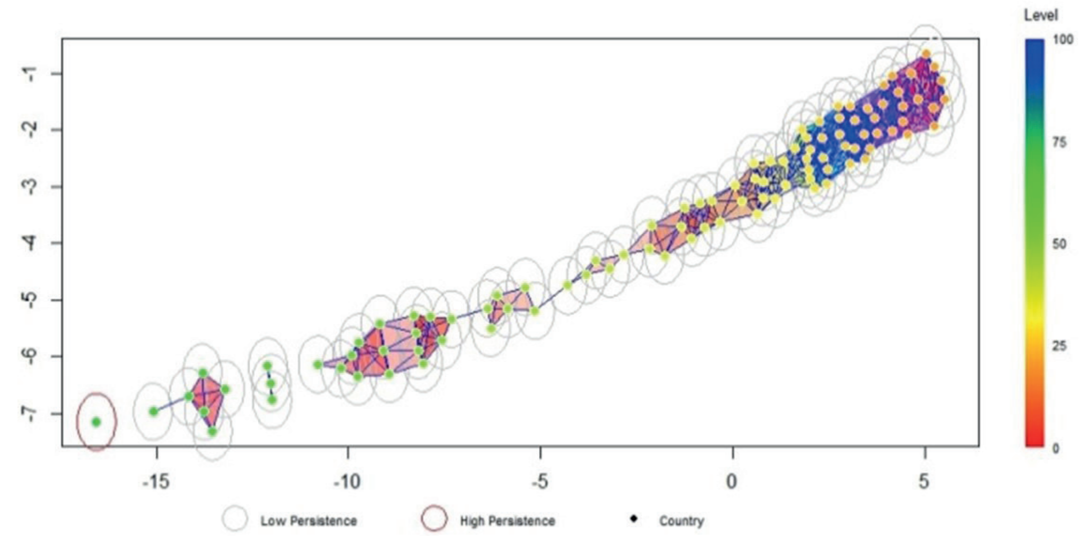

Figure 10. Topological structures captured with Vietoris-Rips complexes on the spatial distribution of the GII 2023 for the 100 most innovative countries (ε = 1).

In 2022, six connected components were observed in the innovation network structure. The principal component, situated in the lower right quadrant, encompassed nations with Global Innovation Index (GII) values ranging from 20 to 40 (orange to yellow spectrum), representing economies such as El Salvador (100), Senegal (99), and Ecuador (98). Two notable secondary components, both displayed in green tones, were positioned in the central area (countries with values between 45 and 55) including Belgium (26), Italy (28), and Spain (29), and in the upper left region (values between 55 and 65) comprising countries such as the United States (2), Sweden (3), and the United Kingdom (4). Additionally, two isolated single-point components were identified, one in the central area and another in the upper left quadrant, the latter exhibiting a relatively high value corresponding to Switzerland (1).

In 2023, the network topology simplified to four connected components. The principal component expanded significantly, extending from the upper right quadrant to the lower central area, with GII values between 23 and 45 and a color gradient including orange, yellow, and light green, incorporating countries such as the Philippines (99), Paraguay (98), as well as medium-innovation economies like Hungary (35) and Poland (41). Two smaller components, each containing 5 to 6 nodes, were located in the lower left quadrant, representing high-innovation economies such as Germany (8), Denmark (9), and France (14). A single isolated component, colored green with a relatively high value, was positioned at the edge of the point cloud, corresponding to Switzerland (1), which maintains its distinctive innovation leadership.

This topological evolution suggests a consolidation and expansion of the main cluster of innovative countries in 2023, absorbing portions of the secondary clusters observed in 2022. The shift toward higher values (from 20-40 in 2022 to 23-45 in 2023) indicates a general improvement in innovation levels. The reduction in the number of connected components (from six to four) and the expansion of the principal component suggest greater cohesion among countries in terms of their innovation capabilities.

However, the persistence of smaller and isolated components in the higher value range indicates the existence of "innovation islands," essentially countries that maintain distinctively high innovation levels compared to the main cluster. This phenomenon signals the continuation of gaps in the global innovation landscape, despite the general trend toward greater homogeneity.

These topological changes reflect a complex dynamic in the global innovation ecosystem, characterized by general progress, greater integration among the majority of countries, and the persistence of clearly differentiated innovation leaders, with significant implications for innovation policies and economic development strategies at the global level.

Multiscale connected components with ε = 1.5 for the GII 2022 and 2023

Figures 11 and 12 show the topological structures captured with Vietoris-Rips complexes with ε = 1.5 from the GII for 2022 and 2023 for the 100 most innovative countries, respectively.

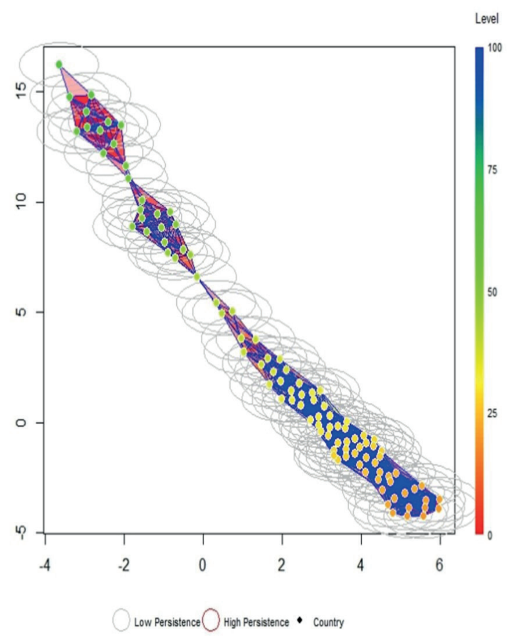

Figure 11. Topological structures captured with Vietoris-Rips complexes on the spatial distribution of the GII 2022 for the 100 most innovative countries (ε = 1.5).

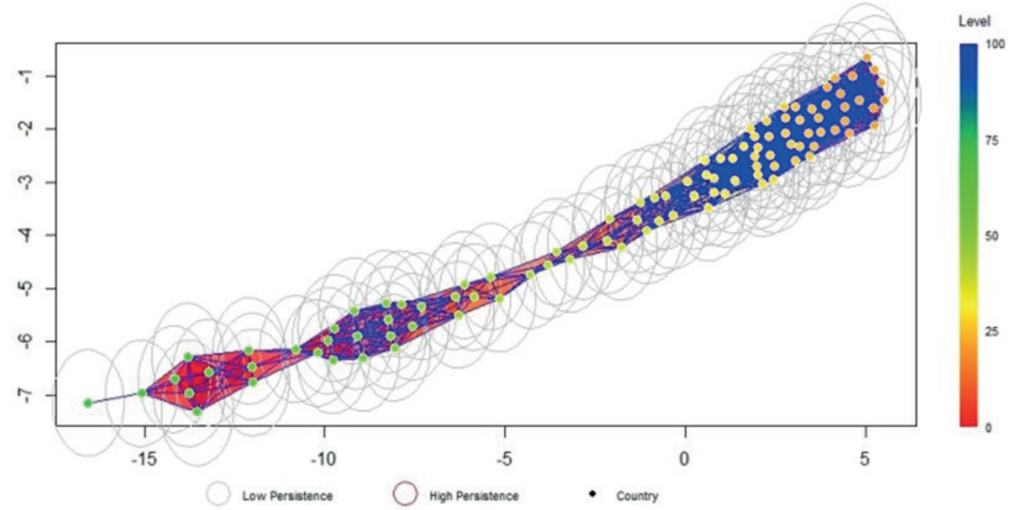

Figure 12. Topological structures captured with Vietoris-Rips complexes on the spatial distribution of the GII 2023 for the 100 most innovative countries (ε = 1.5).

In both years, a single connected component is observed, indicating continuity in the global innovation landscape. This finding is significant because it demonstrates that, at this scale of analysis, no completely isolated groupings exist among countries. Even Switzerland (1), which appeared as an isolated component in analyses with lower ε values, now forms part of the global connected structure.

The color gradient, ranging from orange to light green, suggests a smooth transition in innovation levels without significant discontinuities. In 2022, this gradient flows from countries such as El Salvador (100) and Senegal (99) to Switzerland (1), while in 2023 the transition extends from Kenya (100) and the Philippines (99) to Switzerland (1), maintaining a similar progression.

The stability in the topological structure underscores a gradual evolution in global innovation capacity, without drastic changes in the short term. This indicates that, although there are differences in specific GII values between the two years, the fundamental relational structure of global innovation remains consistent.

This analysis with ε = 1.5 complements previous findings with lower ε values, demonstrating that at broader scales, the global innovation ecosystem functions as an interconnected continuum. Despite differences in innovation levels, there exists a "topological path" connecting all countries.

The persistence of this single connected structure in both years reinforces the concept that global innovation, although uneven, forms an integrated ecosystem where capabilities and knowledge can potentially flow between different regions and economies, regardless of their position in the ranking.

Analysis

The multi-scale connected component analysis for the GII 2022 and 2023, employing persistent homology with Vietoris-Rips complexes, reveals patterns in the evolution of the global innovation landscape. As the parameter ε increases from 0.3 to 1.5, a transformation in the topological structure of the data is observed, providing a detailed insight into the relationships between countries in terms of their innovation capabilities.

At ε = 0.3, the structure is highly fragmented, with 13 components in 2022 and 14 in 2023. This fragmentation reflects detailed differences between countries, capturing nuances in their innovation levels. Countries cluster into multiple small groups, with “innovation islands” such as Switzerland (1), United States (2), and Sweden (3) appearing as isolated components or in small groups separated from the rest. Developing economies such as El Salvador (100) and Senegal (99) in 2022, or Kenya (100) and Philippines (99) in 2023, form part of distinct components, evidencing the significant heterogeneity of the global innovation landscape.

When ε increases to 1, a simplification of the structure is observed. In 2022, 6 components are identified, while in 2023 this number decreases to 4. This consolidation suggests a convergence in innovation levels among groups of countries. Notably, the principal component expands, covering higher values (from 20-40 in 2022 to 23-45 in 2023), indicating a general improvement in innovation levels.

The persistence of smaller and isolated components in the higher value range signals the continuation of gaps and distance in the global innovation landscape. For example, Switzerland (1) continues to appear as an isolated component in both years, highlighting its exceptional position as an innovation leader.

When ε reaches 1.5, complete unification into a single connected component is achieved for both years. This unification suggests an underlying continuity in the global innovation spectrum and the absence of completely isolated groups at this level of analysis. The observed color gradient, ranging from orange to light green, indicates a smooth transition in innovation levels without significant discontinuities. Despite substantial differences in innovation capabilities, there exists a topological path connecting all countries in the ranking, from the most innovative to the least innovative.

The convergence dynamics observed between 2022 and 2023, particularly evident with ε = 1, suggest a process of convergence among countries.

A notable aspect resulting from the topological data analysis is its ability to detect critical transitions in the data structure. The transition observed between ε = 1 and ε = 1.5, where the system shifts from multiple components to a single connected component, represents a critical threshold in the global innovation ecosystem. This transition point can be interpreted as the level at which differences between countries, although present, are not sufficient to create completely isolated groups. This transition may also indicate an increase in the resilience of the global innovation ecosystem, as it implies greater interconnection and potential for collective recovery in the face of challenges. However, it is important to note that the persistence of isolated points with high values across all scales indicates that some countries maintain a distinctive leadership position in innovation.

3.5 Discussion

In the last decade, studies on the Global Innovation Index (GII) have undergone a notable methodological diversification, transcending traditional statistical approaches. Our research on Topological Data Analysis (TDA) applied to the GII contributes to this emerging trend of adopting alternative methodologies that enable the examination of previously unexplored dimensions of national innovation systems. While previous studies have primarily utilized conventional statistical methods and regression analysis to evaluate innovation performance, topological analysis offers a complementary perspective that reveals underlying structural patterns that other methods fail to capture.

Unlike the PCA-DEA approach employed by Alqararah (2023), which focuses on evaluating the robustness of the GII ranking through dimensionality reduction techniques and relative efficiency, our topological analysis examines the relational structure of the global innovation ecosystem at different scales. While Brás's (2023) longitudinal work provides a valuable historical database of GII pillars, and Bate et al.'s (2023) comparative analysis identifies key determinants of innovation performance according to income levels, these methodologies, although rigorous, present limitations for visualizing and analyzing critical transitions in the structure of the global innovation ecosystem.

Traditional methodologies, including regression analysis and ANOVA as employed by Bate et al. (2023), are effective for identifying determinant factors and differences between groups of countries, but lack the capacity to detect critical transition thresholds where the innovation system undergoes qualitative reorganizations. TDA complements these approaches by revealing how the topological structure of the innovation ecosystem evolves at different proximity scales, providing crucial information about group formation and topological distances between countries, allowing for the identification of specific opportunities for strategic interventions in innovation policy.

The comparative analysis of the GII for 2022 and 2023, utilizing graph visualization techniques and topological data analysis deployed in this work, reveals important patterns in the evolution of the global innovation landscape. The insights obtained regarding global innovation dynamics can be considered a contribution to the existing literature on country innovation systems.

Topological analysis shows fragmentation with ε = 0.3, resulting in 13 components in 2022 and 14 in 2023. This fragmentation reflects the diversity in innovation levels among countries, where nations such as Switzerland (1), United States (2), and Sweden (3) form distinct components from countries like El Salvador (100), Senegal (99), and Ecuador (98). This observation relates to the findings of Güell et al. (2022) on regional innovation gaps. Huarng and Yu (2023) highlight the structural complexity of the GII, with its multiple layers of variables divided into input and output sub-indices, presenting methodological scenarios in the evaluation of global innovation.

The reduction in components from 6 to 4 when ε increases to 1 between 2022 and 2023 suggests a consolidation in relative innovation levels. This finding can be related to observations from the GII 2023 on trends in middle and low-income economies. Particularly, the expansion of the principal component to include higher values (23-45 in 2023 compared to 20-40 in 2022) indicates improvements in countries such as Brazil (which rose from position 52 to 49), Chile (50), and Mexico (which fell from 56 to 58). Nasir and Zhang (2024) note that efficiency in implementing innovations plays a role in a country's overall innovative performance, which could influence these observed changes.

The persistence of Switzerland as an isolated point with high value across all scales underscores its sustained leadership in innovation, even increasing its score from 64.6 in 2022 to 67.6 in 2023. The study by Huarng and Yu (2023) on the Qualitative Structural Association of the GII suggests that innovation performance is associated with a specific combination of factors, excluding market sophistication. This observation raises questions about the interaction between different components of the innovation ecosystem and how these interactions vary among countries with different levels of economic and technological development.

The unification into a single component with ε = 1.5 suggests continuity in the global innovation spectrum, despite visible differences between leaders such as Switzerland, Sweden, and the United States, and emerging economies such as China (12) and India (40), which maintain relatively stable positions in both years.

Nasir et al. (2023) in their article on the Global Innovation Intelligence Index (GIII) introduces a machine learning-based approach to evaluate innovation, particularly in emerging technologies. This method could complement topological analysis and provide additional insights into innovation dynamics in specific high-technology areas.

4. Conclusions

This study tested the practical utility of topological methods for examining the global innovation landscape, revealing both changes and continuities between 2022-2023. The findings demonstrate how the innovation ecosystem evolves toward greater cohesion while maintaining persistent hierarchies, with Switzerland consolidating its leadership and economies such as Brazil, China, and India strengthening their regional positions.

The transition from fragmented components to a connected system confirms the interrelated nature of the global innovation landscape, despite significant gaps between leaders and followers.

The Topological Data Analysis (TDA) applied to the Global Innovation Index reveals structural patterns that traditional methods fail to capture. Our results identify critical transition thresholds ($\varepsilon = 1$) where the global innovation ecosystem experiences significant reorganizations, transitioning from a fragmented system to a more cohesive network. The persistence of topological features, such as Switzerland's consistent isolation across different scales and the formation of regional clusters, provides valuable information for policy formulation aimed at reducing innovation gaps.

To enhance global innovation, we propose strategies based on these topological findings: (i) strategic investments focused on the identified transition points, enabling emerging countries to advance between innovation clusters; (ii) optimized international collaborations between topologically proximate countries, maximizing knowledge transfer; (iii) differentiated interventions depending on whether structural barriers or performance gaps are present; and (iv) regional programs adapted to the specific topological characteristics of each country group. The implementation of these strategies, grounded in the topological understanding of the innovation ecosystem, could accelerate global innovative advancement while reducing persistent inequalities between leaders and followers.

The observed trends highlight the importance of emerging technologies and factors beyond economic income in national innovation capabilities. Future research could integrate topological analysis with traditional and contractual statistical techniques to understand long-term innovation trajectories.

This study contributes to the literature by providing a complementary methodological perspective, revealing groupings among countries, transcending traditional classifications, and offering new perspectives for innovation policies that consider this inherent complexity of global innovation ecosystems.

Authors' Contribution

Rolando Ismael Yépez: Conceptualization, Methodology, Investigation, Formal analysis, Project administration, Supervision, Validation, Writing - original draft, Writing - review & editing.

Maricela Fernanda Ormaza: Data curation, Visualization, Resources, Investigation, Validation, Writing - review & editing.

Ethical implications

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